

## Nonlinear Landau damping of electrostatic solitary waves in a quantum plasma

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## Abstract

Landau damping is one of the most fundamental phenomena of waves in plasma physics. Landau damping occurs when the particle's velocity approaches the phase (or group) velocity of the wave and there occurs an energy exchange between them. Such collisionless damping was first theoretically predicted by Landau [1] and later experimentally verified by Malmberg and Wharton [2]. Since then, Landau damping of electrostatic waves in plasmas has been a topic of important research [3-7]. However, in most of these investigations, Landau damping has been considered classically, i.e., using the Vlasov-Poisson system and/or limited to the linear theories. When quantum effects are included, there appears a new length scale and new coupling parameter, as well as new collective modes for which new processes come into play. For example, the modifications of the linear Landau damping of electrostatic waves by the effects of arbitrary degeneracy of electrons [3] and the influence of linear Landau damping on nonlinear waves in quantum plasmas [8].

Motivated by the theoretical and experimental investigations of Landau damping, Ott and Sudan [3] first theoretically investigated the effects of linear Landau damping on the nonlinear propagation of ion-acoustic solitons in electron-ion plasmas through the description of Korteweg de-Vries (KdV) equations, and on the assumption that particle's trapping time is much longer than that of Landau damping. However, their theory was limited to the effects of linear electron Landau damping only. Later this theory was modified by Vandam and Taniuti [4] to include ion Landau damping. This theory has been recently advanced by Barman and Misra [5] in the context of semiclassical plasmas. They investigated the effects of linear Landau damping on the propagation of solitary waves in semiclassical electron-ion plasmas. However, the nonlinear Landau damping of solitary waves in the context of KdV equation has not yet been investigated neither in classical nor in quantum plasmas.

In this work, we investigate the nonlinear Landau damping of electrostatic solitary waves in a fully degenerate quantum plasma. Starting from the Wigner-Moyal equation coupled to the Poisson equation, and using the multiple scale expansion technique (MST), we show that the evolution of the solitary waves is governed by the following KdV-like equation with nonlocal nonlinearity.

$$\frac{\partial \phi}{\partial \tau} + \alpha \phi \frac{\partial \phi}{\partial \zeta} + \beta \frac{\partial^3 \phi}{\partial \zeta^3} + \frac{\gamma}{\pi} \mathcal{P} \int \frac{\partial}{\partial \zeta} \left\{ \frac{\phi^2(\zeta, \tau)}{\zeta - \zeta'} \right\} d\zeta' = 0, \quad (1)$$

where  $\mathcal{P}$  denotes the Cauchy principal value;  $\zeta$  and  $\tau$  are the stretched coordinates in the MST and  $\phi$  is the small but finite amplitude electrostatic potential. In contrast to classical or semiclassical plasmas (where only linear Landau damping appears), the nonlocal nonlinear term ( $\propto \gamma$ ) appears due to the two-plasmon resonance, where the particle's velocity is shifted by  $\hbar k/m$  (i.e.,  $v_2^{res} = \omega/k + \hbar k/m$ ) due to the plasmon energy and momentum [where  $\hbar$  is the reduced Planck's constant,  $\omega(k)$  is the wave frequency (number) and  $m$  is the particle's mass]. Furthermore, the local nonlinear term ( $\propto \beta$ ) is modified not only by the linear resonance (where  $v_1^{res} = \omega/k + \hbar k/2m$ ) but also by the two-plasmon resonance. The effects of these plasmon resonances on electrostatic solitary waves are studied and the applications of our results to dense plasmas are discussed.

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