

Interactions of positron acoustic solitary waves and phase shifts in multi-component plasma

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It is well established that the electron-positron-ion (epi) plasmas are existed in astrophysical, space and laboratory plasmas for understanding the physical issues involved. Recently, Alam et al. [1] have studied the positron acoustic (PA) Korteweg-de Vries (KdV) solitary waves (SWs), modified KdV (mKdV) SWs, Gardner SWs, and double layers (DLs) in the epi plasmas composing immobile positive ions, mobile cold positrons, and superthermal hot electrons and hot positrons. But, the interactions with non-linear waves, including resonances, carry important physical phenomena as observed in space plasmas [2, 3]. This work investigates the interactions of PA waves (PAWs) in unmagnetized plasmas consisting of immobile positive ions, mobile cold positron, and superthermal hot positrons, and hot electrons. The normalized governing equations can be written [1] as

$$\frac{\partial n_{pc}}{\partial t} + \frac{\partial}{\partial x}(n_{pc} u_{pc}) = 0, \quad \frac{\partial u_{pc}}{\partial t} + u_{pc} \frac{\partial u_{pc}}{\partial x} = -\frac{\partial \phi}{\partial x}, \quad \frac{\partial^2 \phi}{\partial x^2} = -n_{pc} - \mu_{ph} \left(1 + \frac{\sigma_1 \phi}{\kappa_p \frac{3}{2}}\right)^{-\kappa_p + \frac{1}{2}} + \mu_e \left(1 - \frac{\sigma_2 \phi}{\kappa_e \frac{3}{2}}\right)^{-\kappa_e + \frac{1}{2}} - \mu_i.$$

Here, n_{pc} is the cold positron number density normalized by its equilibrium value n_{pc0} , u_{pc} is the cold positron fluid speed normalized by $C_{pc} = (k_B T_{ef} / m_p)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_{ef} / e$, $\sigma_1 = T_{ef} / T_{ph}$, $\sigma_2 = T_{ef} / T_e$, $\mu_{ph} = n_{ph0} / n_{pc0}$, $\mu_e = n_{e0} / n_{pc0}$, $\mu_i = n_{i0} / n_{pc0}$, $T_{ef} = T_e T_{ph} / (\mu_e T_{ph} + \mu_{ph} T_e)$ is the effective temperature, k_B is the Boltzmann constant, m_p is the positron mass and e is the magnitude of electron charge. The time t is normalized by $\omega_{pc}^{-1} = (m_p / 4\pi n_{pc0} e^2)^{1/2}$ and the space x is normalized by the Debye length $\lambda_{Dm} = (k_B T_{ef} / 4\pi n_{pc0} e^2)^{1/2}$. To study the solitons collision, two-sided KdV and mKdV are derived using the extended PLK method [2]. The stationary solutions of the KdV equations is obtained as $\phi_\xi^{(1)} = \phi_0 \text{sech}^2 \left\{ \frac{(\xi - U_0 \tau)}{W_0} \right\}$

$\phi_\eta^{(1)} = \phi_0 \text{sech}^2 \left\{ \frac{(\eta + U_0 \tau)}{W_0} \right\}$ and the corresponding phase shifts after weak head-on collision between the KdV solitons $\nabla P_0 = -2\varepsilon^2 \frac{D}{C} \phi_0 W_0$, $\nabla Q_0 = 2\varepsilon^2 \frac{D}{C} \phi_0 W_0$, where $\phi_0 = (3U_0/A)$ and $W_0 = \sqrt{(4B/U_0)}$ are the amplitude and width both of the right and left moving solitary waves in their initial positions, U_0 is the constant velocity of the reference frames, $A = \left[\frac{3}{2\lambda_p^3} + R_2 \lambda_p^3 \right]$,

$$B = \frac{\lambda_p^3}{2}, \quad R_2 = \left\{ \frac{\mu_{ph} (\kappa_p - \frac{1}{2}) (\kappa_p + \frac{1}{2}) \sigma_1^2}{2(\kappa_p - \frac{3}{2})^2} - \frac{\mu_e (\kappa_e - \frac{1}{2}) (\kappa_e + \frac{1}{2}) \sigma_2^2}{2(\kappa_e - \frac{3}{2})^2} \right\},$$

$C = 2\lambda_p$, $D = \left[\frac{1}{2\lambda_p} - R_2 \lambda_p^3 \right]$. Besides, the dispersion coefficient (B) of the coupled KdV equations is always positive. Therefore, the PAWs are obtained compressive for $A > 0$ and rarefactive for $A < 0$ depending on the plasma parameters. It is clearly seen that the amplitude of the KdV solitons and phase shift approaches to infinity when $A \rightarrow 0$ which breaks down the validity of the reductive perturbation technique. In such situation, two-sided mKdV equations are obtained to study the head-on collision between the solitons and phase shift around the critical values (say $\mu_{ph} = \mu_c \cong 0.119$ obtained from $A(\mu_{ph} = \mu_c) = 0$ for the values of the plasma parameters, that is $\mu_e = 0.2$, $\sigma_1 = 1$, $\sigma_2 = 0.5$, $\kappa_p = 3$ and $\kappa_e = 1.8$ [1]). The stationary solutions of mKdV equations can be written as $\phi_\xi^{(1)} = \phi_1 \text{sech} \left\{ \frac{(\xi - U_0 \tau)}{W_1} \right\}$, $\phi_\eta^{(1)} = \phi_1 \text{sech} \left\{ \frac{(\eta + U_0 \tau)}{W_1} \right\}$ and their corresponding phase shift after weak head-on collision between two same amplitudes oppositely propagating mKdV solitons as $\nabla P_0 = -2\varepsilon^2 \frac{D^*}{C} \phi_1 W_1$, $\nabla Q_0 = 2\varepsilon^2 \frac{D^*}{C} \phi_1 W_1$, where $\phi_1 = (\sqrt{6U_0/\alpha B})$ and $W_1 = \phi_1 \sqrt{(\alpha/6)}$ are the amplitude and width both of the right and left moving mKdV solitary waves in their initial positions, $\alpha = \left[\frac{15}{2\lambda_p^6} - 3R_3 \right]$, $D^* = \left[3R_3 - \frac{1}{2\lambda_p^6} \right] B$,

$$R_3 = \left\{ \frac{\mu_{ph} (\kappa_p - \frac{1}{2}) (\kappa_p + \frac{1}{2}) (\kappa_p + \frac{3}{2}) \sigma_1^3}{6(\kappa_p - \frac{3}{2})^3} + \frac{\mu_e (\kappa_e - \frac{1}{2}) (\kappa_e + \frac{1}{2}) (\kappa_e + \frac{3}{2}) \sigma_2^3}{6(\kappa_e - \frac{3}{2})^3} \right\}.$$

It is found that the presence of superthermal hot positrons and hot electrons significantly modify the soliton collision and phase shifts in the plasmas. The critical values for hot- and cold positrons also play a vital role in the formation of only compressive PA mKdV SWs. It can be conclude that the energy absorption appears by the solitons collision without changing the shape and velocity, thus the energy absorption enhances (suppresses) smoothly with the enhancement of plasma parameters for the rarefactive (compressive) PA SWs. Furthermore, the system critically changes the polarity, which makes the energy absorption and phase shift have maximum change near the critical values.

References

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