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Extension of gyrokinetic field theory

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The gyrokinetic theory is a basic framework to describe microinstabilities, turbulence, and resultant anomalous transport observed in magnetically confined plasmas. Nonlinear gyrokinetic equations for particle distribution functions were originally derived by recursive techniques combined with the WKB representation [1]. Modern gyrokinetic theories are based on the Lagrangian/Hamiltonian formulation using the Lie transformation to define the gyrocenter coordinates in which the motion equations exactly satisfy Liouville's theorem and invariance of the magnetic moment [2]. Furthermore, in the gyrokinetic field theory [3], gyrokinetic Poisson and Ampère equations for turbulent electromagnetic fields are derived from the Lagrangian variational principle, and Noether's theorem can be naturally applied to prove the energy conservation for the whole system.

The present paper reviews the gyrokinetic field theory which is extended to include effects of the evolution of background fields, Coulomb collisions, and large $\mathbf{E}\times\mathbf{B}$ and toroidal flows on the order of the ion thermal velocity [4]. We consider the toroidally rotating plasma confined in the axisymmetric background magnetic field $\mathbf{B}_0$. The gyrocenter motion equations for the charged particles and the equations for the background and turbulent electromagnetic fields are all derived from the variational principle $\delta \int_{t_1}^{t_2} L\, dt = 0$ where the Lagrangian L for the whole system is written as [4]

$$L = \sum_a \int d^6 Z_0\, D_a(\mathbf{Z}_0, t_0) F_a(\mathbf{Z}_0, t_0) L_a\big[\mathbf{Z}_a(t), \dot{\mathbf{Z}}_a(t)\big] + \int d^3x\, \mathcal{L}_f \, . \qquad (1)$$

Here, $F_a(\mathbf{Z_0}, t_0)$ and L_a represent the particle distribution function at the initial time t_0 and the single-particle Lagrangian for particle species a, respectively, and $\mathcal{L}_f$ is the Lagrangian density associated with electromagnetic fields. The gyrocenter coordinates are denoted by $\mathbf{Z}_a = (\mathbf{X}_a, U_a, \mu_a, \xi_a)$ where $\mathbf{X}_a$, U_a, μ_a, and ξ_a are the gyrocenter position, parallel velocity (observed from the toroidally rotating frame), magnetic moment, and gyrophase angle, respectively.

The gyrokinetic Boltzmann equation for the gyrocenter distribution function F_a is written as

$$\left(\frac{\partial}{\partial t} + \frac{d\mathbf{Z_a}}{dt}\cdot\frac{\partial}{\partial \mathbf{Z}}\right) F_a(\mathbf{Z}, t) = \sum_b \langle C_{ab}[F_a, F_b]\rangle_\xi \, , \qquad (2)$$

where $d\mathbf{Z}_a/dt$ is given from the gyrocenter motion equations, $\langle\cdots\rangle_\xi$ is the gyrophase average, F_a is assumed to be independent of the gyrophase ξ, and $C_{ab}[F_a, F_b]$ represents the rate of change F_a due to the Coulomb collisions between particle species a and b. For the collisionless case, $C_a \equiv \sum_b C_{ab} = 0$, Eq.(2) reduces to the gyrokinetic Vlasov equation, for which Noether's theorem can be applied to derive conservation laws of energy and toroidal momentum from symmetry properties. However, even for the collisional case, $C_a \neq 0$, we can still derive the energy and toroidal momentum balance equations from Noether's theorem modified using the correspondence relation between $\partial F_a^V/\partial t$ and $\partial F_a/\partial t - C_a$, where F_a^V and F_a represent the solution of Eq. (2) for $C_a = 0$ and that for $C_a \neq 0$, respectively [5].

Here, we follow Burby *et al.* [6] and use the Poisson bracket $\{\cdot, \cdot\}$ to write the collision operator as

$$C_{ab}[F_a, F_b] = -\alpha_{ab} \sum_{i=1}^{3} \{x_{ai}, \gamma_i^{ab}\} \, , \qquad (3)$$

where x_{ai} and γ_i^{ab} are the ith Cartesian components of the particle position vector and the vector defined in [4,6], respectively, and $\alpha_{ab} \equiv 2\pi e_a^2 e_b^2 \ln\Lambda$. It is verified by using the collision operator defined in Eq.(3) that the total energy and toroidal momentum conservation laws are satisfied by the gyrocenter distribution functions and the electromagnetic fields which are obtained from solving the gyrokinetic system of equations. The derived conservation laws of particles, energy, and toroidal momentum are shown to simultaneously contain all classical, neoclassical, and turbulent transport fluxes which agree with the conventional results obtained from the recursive formulations. It is emphasized that the background $\mathbf{E}\times\mathbf{B}$ and toroidal flows can be determined by the toroidal momentum conservation law. In addition, discussions are made on the entropy production and on the momentum transport flux (or the viscosity tensor) derived from the invariance of the action integral under an arbitrary spatial coordinate transformation.

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