



Unified N -body description of Debye shielding and Landau damping

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In contrast to conventional wisdom, several fundamental aspects of the kinetic physics of plasmas can be tackled by N -body mechanics. This approach reveals that *Landau damping and Debye shielding come from a cooperative dynamical self-organization process*, produced by the accumulation of *almost independent* Coulomb deflections. Unexpectedly, *shielding and collisional transport are two aspects of the same two-body repulsive/attractive process*.

The N -body description of a plasma is amenable to the same linear approach as the one leading to Landau pole calculation in a Vlasovian frame [1,2]. Approximating the granular N -body velocity distribution function by a spatially continuous one, shows that the electrostatic potential is the *sum over the N particles of their individual shielded Coulomb potentials and of their contributions to the excitation of Langmuir waves*. This is done for a single realization of the plasma by using standard tools of elementary mechanics, calculus and no probabilistic setting.

Since the N -body system is a Hamiltonian one, its Lyapunov exponents come in pairs: a damped eigenmode comes with a companion with the opposite growth rate. Therefore, *Landau damping cannot correspond to a damped eigenmode*, since otherwise its growing companion would be present and would be excited for a typical initial condition. When keeping the granular N -body velocity distribution function, the above approach reveals that *Landau damping is due to a phase mixing à la van Kampen* (section 4.5 of [1] and [3]).

Applying Picard fixed-point iteration technique to the equation of motion of a particle P due to the Coulomb forces from the $N-1$ other ones, reveals that a part of the effect on particle P of another particle P' is mediated by the other $N-2$ particles and reduces the direct part for a single-species plasma in a neutralizing background [1]. Indeed, in the Debye sphere about P' , the part of the global deflection of particles due to P' *reduces the number of particles inside this sphere*, which screens the charge of particle P' out of it, according to Gauss' theorem. In a multi-species plasma, the attractive deflection of charges with opposite signs would have the same effect.

When starting from random particle positions, a plasma period is necessary for shielding to set in, and for the plasma to behave like a dielectric. Paradoxically, *Landau damping turns out to exist because of what is traditionally called 'collisions'*.

The N -body approach has further capabilities. By splitting the set of particles into a bulk, which cannot resonate with Langmuir waves, and a tail, amplitude equations are derived for the Fourier components of the

potential where tail particles provide a source term [1,4]. These equations, together with the equations of motion of the tail particles, provide a *Hamiltonian description of Langmuir waves exhibiting spontaneous emission*. In this approach, for a Landau-unstable wave, the phase mixing of van Kampen-like eigenmodes cancels the contribution of the companion of the unstable one [3,4]. This proves the Vlasovian limit to be very singular. The bulk-tail description provides also nonlinear effects like damping with trapping [4,5], and the decoupling of the waves from the strongly chaotic motion of particles when there is a plateau in the particle velocity distribution [6].

An N -body model where electrons interact through their Debye shielded Coulomb potential, enables the *calculation of collisional transport describing correctly all impact parameters b* , with a convergent expression reducing to Rutherford scattering for small b [7]. This derivation explains why a two-body calculation yields a correct estimate of collisional transport, while most of this transport is due to the simultaneous action of many particles with impact parameters between the inter-particle distance and the Debye length.

All the above results show that N -body dynamics, which is the ultimate reference in plasma textbooks, now is also a practical tool: Laplace's dream was not a mere utopia, *since classical mechanics can genuinely describe and explain non-trivial aspects of the macroscopic dynamics of a many-body system*.

References

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