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Helically deformed MHD equilibrium as lower-energy state via simulated annealing

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A new method for calculating magnetohydrodynamics (MHD) equilibrium, called simulated annealing, has been developed recently. The method is based on Hamiltonian formulation of dynamics of ideal fluids including ideal MHD. The dynamics of ideal fluids is described by the Hamiltonian and the non-canonical Poisson brackets [1, 2]. Namely, the system evolves according to the vector field defined by the functional derivative of the Hamiltonian with an operation by the Poisson bracket. The anti-symmetry of the Poisson bracket provides energy conservation. Interestingly, the Poisson bracket also provides Casimir invariants that are defined as the null space of the Poisson bracket. Therefore the system evolves on a surface specified by its energy and the Casimir invariants in the corresponding phase space. The surface specified by the values of the Casimir invariants is called the Casimir leaf. An energy extremum on the Casimir leaf gives an equilibrium[3, 4].

Here, let us consider an artificial dynamics derived from the original ideal fluid system; the Poisson bracket is operated twice in the artificial dynamics. Then the energy of the system changes monotonically while preserving the Casimir invariants. Then the artificial dynamics will lead to the energy extremum on the Casimir leaf. Therefore this artificial dynamics can be used as a new method for calculating an equilibrium of the ideal fluids. The equilibrium obtained by this method is characterized by the values of the Casimir invariants. This idea was applied for two-dimensional vortical motion of neutral fluids in [5, 6, 7]. The artificial dynamics derived by double operation of the Poisson bracket is an example that realizes monotonic change of the energy. More advanced types of artificial dynamics have been also developed and demonstrated for the neutral fluids [8], that are termed simulated annealing (SA).

The same idea also applies MHD. So far, we have successfully obtained equilibria, or stationary states, of two-dimensional low-beta reduced MHD[9] by the SA in a rectangular domain with doubly periodic boundary conditions[10]. We have also developed a method to specify the values of the Casimir invariants before the SA calculation; those values are conserved during the SA calculation[11]. Furthermore, we have extended the code for a cylindrical plasma, where the interior of the plasma can deform in three-dimensions although the plasma shape is fixed to a cylinder[12]. The implicit Runge-Kutta method was recently implemented as the time stepper, which is symplectic and time-reversal symmetric. The implicit equation is solved by the Newton method and the generalized minimal residual (GMRES) method. Then the numerical stability is significantly improved[13].

Because the SA decreases the energy of the system monotonically, the obtained equilibrium has a lower energy compared to the initial condition. Therefore, if we start the SA from a cylindrically symmetric state plus a tiny helical perturbation, the system goes to a helically deformed state if the cylindrically symmetric state is unstable, while the system comes back to the cylindrically symmetric state if it is stable. An example of such a helically deformed equilibrium is shown in [12], where magnetic islands exist in the equilibrium. We have also obtained some simulation results where the tiny perturbation dies away when the cylindrically symmetric state is stable. Such numerical results will be shown together with the helically deformed equilibria in the presentation. Especially the helically deformed equilibria of the internal kink type will be presented. Moreover, the effect of the initial condition on the final equilibrium will be also examined, because we have many choices of the tiny helical perturbation that are on the same Casimir leaf.

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