

## Forced-dissipative turbulence governed by generalized two-dimensional fluid systems

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The present talk introduces the wavenumber space dynamics of forced-dissipative turbulence governed by the generalized two-dimensional (2D) fluid systems developed in these 15 years.

It is well known that forced-dissipative 2D turbulence governed by the Navier-Stokes (NS) equation has peculiar characteristics compared to the three-dimensional NS turbulence. If the system is forced at narrow wavenumbers around  $k_f$ , the energy inertial range is formed in the lower wavenumber side of  $k_f$ . In the energy inertial range, the energy is transferred toward lower wavenumbers while the enstrophy (the spatial integration of squared vorticity) is not at all transferred, and the energy spectrum has the form  $E(k) \sim k^{-5/3}$ , where  $k$  indicates the wavenumber. In contrast, the enstrophy inertial range is formed in higher wavenumber side of  $k_f$ . In the enstrophy inertial range, the enstrophy is transferred toward higher wavenumbers while the energy is not at all transferred, and the energy spectrum has the form  $E(k) \sim k^{-3}$ . The existence of two inertial ranges is attributed from the existence of two inviscid invariants, energy and enstrophy, of 2D NS equations.

The generalized 2D fluid systems are governed by the nonlinear advection equation for an advected scalar  $q$  due to incompressible flows. Hereafter, we shall simply call  $q$  as vorticity, although  $q$  is not the vorticity in the regular sense. The generalized 2D fluid systems are characterized by the relation between  $q$  and the stream function  $\varphi$ :  $q = -(\Delta)^{\alpha/2}\varphi$ . It was originally introduced by Pierrehumbert *et al.* (1994) as a tool for studying the effects of spectral non-locality on 2D NS turbulence. These systems also include some realizable members of geophysical fluid and plasma physics systems. For  $\alpha=2$ , the governing equation of these systems reduce to the vorticity equation of the 2D NS system. Furthermore, the generalized 2D fluid systems has also two inviscid invariants, energy and enstrophy, like as 2D NS system. The parameter  $\alpha$  controls the non-locality of the wavenumber dynamics. Turbulence governed by the generalized 2D fluid systems, the so-called  $\alpha$ -turbulence, has been actively investigated in these 15 years.

Since the parameter  $\alpha$  is included in the governing equation of the systems, turbulence properties such as power-law exponent of the enstrophy spectrum is expected to depend on  $\alpha$ . In this sense, the power-law exponents of the enstrophy spectrum have been actively investigated up to now. Note that the relation between the energy and enstrophy spectra is given by  $Q(k)=k^\alpha E(k)$ .

In the infrared range ( $k \rightarrow 0$ ), the non-local interactions in the wavenumber space are important and the enstrophy

spectrum has the form  $Q(k) \sim k^5$  irrespective of  $\alpha$ . These properties have been predicted by the eddy damped quasi-normal Markovian (EDQNM) approximation equations, which is one of the closure equations, and also confirmed by direct numerical simulations (DNSs) (Iwayama and Watanabe, 2014).

In the enstrophy inertial range, the enstrophy spectrum exhibits the transition from  $Q(k) \sim k^{-(7-2\alpha)/3}$  to  $Q(k) \sim k^{-1}$  at  $\alpha=2$ . The transition of the power-law exponent has been confirmed by DNS (Watanabe and Iwayama, 2004; 2007). The origin of the transition of the power-law exponent is attributed from the transition of the contribution of non-local interactions in the wavenumber space at  $\alpha=2$ . This is confirmed by phenomenological arguments and analytical calculations of the EDQNM equations (Watanabe and Iwayama, 2004; Iwayama and Watanabe, 2016).

In contrast to the infrared and enstrophy inertial ranges, the wavenumber dynamics in the energy inertial range has been understood yet not. DNSs indicate that the enstrophy spectrum in the energy inertial range has the form  $Q(k) \sim k^{-(7-4\alpha)/3}$  for  $\alpha < 5/2$ , while  $Q(k) \sim k$  for  $\alpha > 5/2$  (Burgess and Shepherd, 2013). That is, the power-law exponent exhibits transition at  $\alpha=5/2$  and the enstrophy spectrum seems to be the enstrophy equipartition spectrum. However, there is no phenomenology to explain this transition and the analysis of the EDQNM fails the prediction of the power-law exponent. That is, the investigation of turbulence properties in the energy inertial range of  $\alpha$ -turbulence has been unsolved yet and one of the challenging areas of  $\alpha$ -turbulence studies.

### References

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