Zonal radial electric field (REF) in a tokamak is important in regulating the plasma turbulence and confinement improvement, such as the transition to the H-mode. The zonal REF can be argued based on the ion radial force balance equation in the cylindrical geometry

\[
ne(E_r + u_\phi B_r - u_\psi B_p) - \partial_r p_i = 0.
\]

Here, \( n \) and \( e \) are the ion density and charge, respectively. \( E_r \) is the zonal REF, \( u_\phi \) and \( u_\psi \) are the poloidal and toroidal flow, respectively. \( B_p \) and \( B_r \) are the poloidal and toroidal magnetic field, respectively. \( p_i \) is the ion pressure. By introducing the fluid transport equations, the radial force balance equation can be written as\(^1\)

\[
\partial_t E_r = -B_p \frac{1}{nm_i r} \partial_r (r \Pi_{r\phi, i}) - \frac{1}{ne} \partial_r \left[ r \partial_r \left( r \frac{2}{3} Q_r, i \right) \right] + B_r \frac{1}{nm_i r} \partial_r (r \Pi_{r\phi, i}).
\]

Here, \( m_i \) is the mass, \( \Pi_{r\phi, i} = nm_i \langle \hat{v}_r \hat{v}_\phi \rangle \) is the radial component of the turbulent poloidal Reynolds stress, with \( \hat{v}_r \) and \( \hat{v}_\phi \) the fluctuating components of ion fluid radial velocity and toroidal velocity in the torus, respectively, and \( \langle \cdot \rangle \) denotes the ensemble average. \( \Pi_{r\phi, i} = nm_i \langle \hat{v}_r \hat{v}_\phi \rangle \) is the radial component of the turbulent poloidal Reynolds stress, with \( \hat{v}_r \) and \( \hat{v}_\phi \) the fluctuating components of ion fluid poloidal velocity. \( Q_r, i = \frac{3}{2} \langle \hat{v}_r \hat{p}_i \rangle \) is the radial component of turbulent energy flux, with \( \hat{p}_i \) the fluctuating component of ion fluid pressure. When the turbulent toroidal Reynolds stress and the turbulent energy flux are neglected, Eq. (2) reduces to the previous nonlinear turbulent poloidal Reynolds stress-driving model\(^2\)

\[
\partial_t E_r = B_T \frac{1}{nm_i r} \partial_r (r \Pi_{r\phi, i}).
\]

In the toroidal geometry, Itoh \ et \ al. pointed that the REF driven by the turbulent poloidal Reynolds stress should be modified\(^3\) as

\[
\epsilon_r \partial_t E_r = B_T \frac{1}{nm_i r} \partial_r (r \Pi_{r\phi, i}).
\]

Here, \( \epsilon_r \approx 1 + 2q^2 \) is the neoclassical polarization enhancement factor, with \( q \) the safety factor. Using the gyrokinetic theory, Rosenbluth-Hinton found \( \epsilon_r \approx 1 + 1.6q^2 / \sqrt{\epsilon} \), with \( \epsilon \) the inverse aspect ratio\(^4\). However, experimental observations of the turbulent poloidal Reynolds stress on the REF seem ambiguous. The EAST experiment\(^5\) indicates the qualitative agreement with the nonlinear turbulent poloidal Reynolds stress-driving model, while the observation in the JFT-2M tokamak do not\(^6\).

In this work, the turbulent toroidal Reynolds stress and the turbulent energy flux are added into the nonlinear source term in the Friemann-Chen’s gyrokinetic equation. The result shows that besides the enhancement of the REF, the turbulent toroidal Reynolds stress and the turbulent energy flux can also be enhanced due to the toroidal effect\(^1\)

\[
\epsilon_r \partial_t E_r = -\epsilon_r B_p \frac{1}{nm_i r} \partial_r (r \Pi_{r\phi, i})
\]

\[
-\epsilon_r \frac{1}{ne} \partial_r \left[ \frac{1}{r} \partial_r \left( r \left( \frac{2}{3} Q_r, i \right) \right) \right].
\]

As an example, we also generalize Rosenbluth’s problem of residual zonal flow by adding an impulse of ion heating to the impulse source, the solution based on the present theory is readily found

\[
k^2 \delta \phi_k = \frac{1}{\epsilon_r} k^2 \delta \phi_k (0) + ik u_\zeta, i k (0) B_p
\]

\[
-\frac{k^2}{\epsilon} T (0).
\]

Here, \( k \) is the zonal radial wavenumber. The result indicates that although the initial impulse of ion density is significantly shielded by the neoclassical effect, the initial impulses of both the ion pressure and the ion parallel momentum are not.

The proposed theory may resolve the controversial issues on the Limit-Cycle-Oscillation dynamics raised by the recent tokamak L-H transition experiments.

References


Figure. Terms of the radial force balance equation in torus computed by gyrokinetic global simulation in the nonlinear stage. Only the \( u_\zeta B_p \) terms is involved in V cross B term.