The radial electric field plays an important role in the L-H transition. In the early days, Diamond pointed out that the radial electric field can be driven by the turbulent poloidal Reynolds stress (PRS) \( \Pi_{\rho \phi} \), which can be written in the cylindrical geometry as

\[
\partial_t E_r = B_T \partial_r \Pi_{\rho \phi}.
\]

(1)

Here, \( \Pi_{\rho \phi} = (\vvec_r \cdot \vvec_{\phi}) \) with \( \vvec_r \) and \( \vvec_{\phi} \) the radial and poloidal component of the fluctuated \( \vvec \times \vvec \) drift velocity, respectively. \( \langle \cdot \rangle \) denotes the ensemble average. Later, Itoh pointed out that the radial electric field driven by the turbulent PRS should be modified due to the toroidal effect as

\[
\epsilon_r \partial_t E_r = B_T \partial_r \Pi_{\rho \phi}.
\]

(2)

Here, \( \epsilon_r \approx 1 + 2q^2 \) is the neoclassical polarization enhancement factor, with \( q \) the safety factor; Rosenbluth modified this factor to be \( \epsilon_r \approx 1 + 1.6q^2/\sqrt{\epsilon} \), with \( \epsilon \) the inverse aspect ratio. Recently, based on Rosenbluth’s work, Wang pointed out that besides the enhancement of the radial electric field, the turbulent toroidal Reynolds stress \( \Pi_{\rho \phi} \) and the turbulent energy flux \( Q_r \) can also be enhanced due to the toroidal effect

\[
\epsilon_r \partial_t E_r = \epsilon_r \left( -B_T \partial_r \Pi_{\rho \phi} - \frac{2}{3} \partial^2 Q_r \right).
\]

(3)

However, whether the turbulent PRS can be modified due to the toroidal effect in the works shown in Refs. [2-4] is still unknown; the main reason may be that the calculation of the PRS based on the gyrokinetic (GK) equation is not yet clear.

Though the turbulent PRS is widely studied based on the fluid theory, the analysis based on the GK theory is relatively rare. In this work, transport of the poloidal momentum due to the electrostatic turbulence is studied with the GK theory in a slab geometry. By taking the poloidal velocity moment integral of the GK equation, the poloidal momentum equation is derived, which is written as

\[
\partial_t (n v_p) + \partial_r \Pi_{xy} = 0.
\]

(4)

It is found that the poloidal momentum \( n v_p \) contains the contribution from the poloidal component of the \( \vvec \times \vvec \) drift velocity, the polarization drift velocity and the fluid diamagnetic drift velocity. In the PRS \( \Pi_{xy} \), there exists the fluid part and the kinetic modification part. The dominant term in the fluid part is the same as that derived from the fluid theory, which relates to the turbulence radial wavenumber; the kinetic part is due to the correlation between the fluctuated \( \vvec \times \vvec \) drift velocity and the fluctuated diamagnetic drift velocity, and this part relates to the radial inhomogeneity of the turbulence intensity. Since the expression of the transport fluxes relate to the gyrocenter distribution function \( F \), which can be directly obtained from the GK simulation, this work may be a theoretic reference for the analysis of the poloidal momentum transport based on the numerical data. Also, the GK calculation of the poloidal momentum transport may make it possible to study the effect of the PRS on the radial electric field in the toroidal geometry, as is discussed in Ref. [4].

References