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Zonal Flows in Rotating Fluids: Phenomenological Interest and Theoretical Problems

Michio Yamada
Research Institute for Mathematical Sciences (RIMS), Kyoto University
yamada@kurims.kyoto-u.ac.jp

In planets in our solar system, zonal flows are usual phenomena which appear stable for a very long time. A typical example may be the atmospheric motion in Jupiter where we can observe a zonal pattern consisting of brown and white stripes. However, the origin of the zonal flows has long been a subject of discussion. Roughly speaking there appears two different theories; one attributes the zonal pattern to deep thermal convection in the planet, while the other to two-dimensional (2D) dynamics of the surface atmosphere. Here we focus our attention to the latter 2D dynamics, not only because it may be a simplest model for the zonal flows, but also because it includes an interesting theoretical problem of spontaneous pattern formation in fluid turbulence.

Our model system is 2D non-divergent neutral fluid on a rotating sphere, governed by the 2D Navier-Stokes (NS) equation on a rotating 2D unit sphere. We should mention that 2D NS equation is not uniquely defined because the viscous term along a 2D manifold depends on assumptions employed by researchers. Here we take the 2D NS equation with a viscous term which conserves the angular momentum of the fluid motion,

$$\frac{\partial \omega}{\partial t} + J(\psi, \omega) + 2\Omega \frac{\partial \psi}{\partial \phi} = \frac{1}{R}(\Delta + 2)\omega + F$$

where ψ is the stream function, $\omega = \Delta \psi$ the vorticity, Ω the rotation rate of the unit sphere, ϕ the longitude, *R* the Reynolds number, *F* the forcing, Δ the Laplace-Beltrami operator on a unit sphere and J is the nonlinear term.

An artificial but basic question may be the asymptotic behavior of the solution in the presence of a zonal (i.e. eastward or westward) forcing. A simplest zonal forcing may be

$$F = \frac{1}{R}[l(l+1) - 2)Y_l^0(\mu)$$

consisting of a single spherical harmonic function ($\ell \geqq 2$), where μ is the sine of latitude, for which the steady flow is $\psi = Y_\ell^0(\mu)$ consisting of ℓ zonal jets (ℓ-jet flow).

Interestingly, when $\ell=2$, 2-jet flow is proved to be globally stable, i.e. any solution asymptotically converges to the 2-jet flow for arbitrary values of Ω and *R*. This strong stability is, however, not shared by larger values of ℓ, where the ℓ-jet flow at moderate values of Ω becomes linearly unstable for sufficiently large values of *R* [1].

Numerical study of the linear stability of ℓ-jet flows shows that there is an interval [Ω_+, Ω_-], for Ω outside of which the ℓ-jet flow is linearly stable for an arbitrary value of *R*. We expect that this linear stability result may be extended to nonlinear cases (global stability). Actually we have a little extended result for the case in which $\Omega \geqq 0$ (without loss of generality) and the forcing term $F=F(\mu,t)$ is an arbitrary smooth function of μ and time *t*. In this case any solution asymptotically converges to $Z(t)$ for an arbitrary Ω above a rotation rate Ω_0 and an arbitrary *R*, where $Z(t)$ is the *unsteady* but *zonal* flow solution corresponding to the unsteady zonal forcing $F(\mu,t)$ [2].

In Jovian atmosphere, we cannot assume such a purely zonal forcing. As we do not have enough knowledge about the dynamical forcing mechanism, we assume here *F* to be a homogeneous isotropic random forcing, with an expectation that it may model a forcing arising from thermal convection, for example.

Numerical simulation with the random forcing shows at an early stage of time development, a zonal flow pattern emerges with several eastward/westward jets. The zonal jet pattern then persists for some long time, and the 2D model produces a zonal flow stripe pattern similar to that observed in Jovian atmosphere. However, the zonal pattern appears changing slowly as time goes on. In fact much longer numerical simulation shows that in a long course of time development one jet merges with an adjacent jet successively, and the number of jets gradually decreases with the final state consisting of two or three zonal jets. We should remark that the reduction of the number of jets is not observed in the beta-plane turbulence, and is considered to arise from geometrical conditions of 2-sphere, which is not taken into account in the beta-plane approximation [3].

These simulations show the stability or robustness of the jets, but it is not yet clear how and why the zonal flow emerges. It is known that even in freely decaying case with no forcing and numerically vanishing viscosity, westward circumpolar jets still appears, although no strong zonal jet pattern is then observed in mid or low latitudes. This system is governed by nonlinear interactions of Rossby waves, and it is found that most energy is transferred to the zonal modes which have resonance pairs, in spite of the fact that the resonant interaction of the Rossby waves cannot transfer the energy to the zonal modes, which indicates that even non-resonant interaction of the Rossby waves transfer the energy favorably to the zonal flow modes which can be in resonance with other waves [4].

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