Hamiltonian and Metriplectic Descriptions of Plasma and other Matter

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Physical models that describe the dynamics of matter, whether they be discrete, like those for interacting particles or dust, or continuum models, like those for fluids and plasmas, possess structure. The structure may be of *Hamiltonian* type (see [1, 2] for review) and/or posses dissipation and exhibit *metriplectic* structure [3] (see [4] for review). The structure may give rise to conservation laws resulting from Galilean, Poincare, or other invariance, or it may assure the property of entropy production giving relaxation to thermal equilibrium. On a basic level, all structure ultimately arises from an underlying Hamiltonian form that may or may not be maintained in approximations and/or reductions of various kinds.

I will survey the structure and its uses for a variety of models, with an emphasis on general magnetofluid models [5, 6, 7, 8, 9, 10, 11] and Vlasov-Maxwell theory [1, 12]. In particular, I will discuss structure preserving numerical algorithms and how structure can be used to design algorithms for specific purposes [13, 14, 15, 16]. Although symplectic integration has been well studied and widely used for finite-dimensional systems, the preservation of the structure that occurs in continuum models such as extended magnetohydrodynamics with generalized helicities, is considerably more difficult to implement. Progress in developing a discrete version of the Maxwell-Vlasov system that preserves its Hamiltonian structure, and its numerical implementation will be discussed [14].

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