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Vortex reconnection and a finite-time singularity of the Navier-Stokes equations

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Vortex reconnection has been studied intensively as a fundamental process both in classical and quantum turbulence. Inspired by the recent experiment by Kleckner & Irvine on the dynamics of a trefoil knot vortex [1], we first considered a linearized model in which two Burgers-type vortices are driven together by an axisymmetric straining velocity field [2]. With this model, we could demonstrate that the time-scale of reconnection is independent of kinematic viscosity ν in the limit ν -> 0 and that the initial helicity associated with the two Burgers-type vortices decays to zero during the reconnection process.

We then investigated the evolution of an eight-figured vortex using the Biot-Savart model to elucidate the nonlinear effect of vortex-vortex interaction on the process of vortex reconnection [3]. In this model, the closed loop of the vortex is discretized into piecewise linear segments, and the velocity of each segment is calculated by evaluating the Biot-Savart integral as a sum of the interactions from the other segments, which is called as the cut-off method. While the ring parts of the eight-figured vortex move to opposite directions, the center parts deform to nearly an anti-parallel configuration first and then finally collide after producing a pair of cusps. By varying the number of segments systematically, we controlled the resolution of the calculations and showed that the minimum distance of the colliding cusps as a function of time scales as $(t_C-t)^{1/2}$ where t_C is an estimated collision time. Also we verified that the scaling of maximum velocity and the axial stretching rate at the cusps are close to $(t_C-t)^{-1/2}$ and $(t_C-t)^{-1}$, respectively. All these scaling exponents imply a similarity with the scaling proposed by J. Leray for possible self-similar solutions of the Navier-Stokes equations [4],[5].

A scrutiny of the vortex configuration up to just before the time of reconnection suggested that a certain universal geometric shape of vortices is formed during the process as pointed out in the previous paper Kimura & Moffatt (2018) [3]. This idea agrees with the concept proposed by de Waele & Aarts [6]. In the paper, using various initial configurations of two vortex rings, they argued that a symmetric pyramidal structure is (universally) formed before reconnection. By extending their concept, a simple model of vortex reconnection called a tent model is presented, and using this model, some geometric properties associated with vortex reconnection is considered in Kimura & Moffatt (2018) [7].

Extending the configuration of tilted hyperbola to more natural one that has a finite energy, we placed two tilted circular vortex rings of circulation $\pm\Gamma$ and radius R, symmetrically on two planes at angles $\pm\alpha$ to a plane of symmetry $x = 0$. The minimum separation of the vortices, $2s$, and the scale of the core cross-section, δ, are supposed to satisfy the initial inequalities $\delta << s << R$, and the vortex Reynolds number $R_\Gamma = \Gamma/\nu$ is supposed very large. It is shown that the behavior near the points of closest approach of the vortices (the tipping points) is determined solely by the curvature $\kappa(\tau)$ at the tipping points and by $s(\tau)$ and $\delta(\tau)$, where $\tau = (\Gamma/R^2)t$ is a dimensionless time variable. The Biot-Savart law is used to obtain analytical expressions for the rate of change of these three variables, and a nonlinear dynamical system relating them is obtained. The solution of the dynamical system shows a finite-time singularity, but the Biot-Savart law breaks down just before this singularity is realized when $\kappa(\tau)s(\tau)$ and $\delta(\tau)/s(\tau)$ become of order unity [8]. To analyze this phenomenon, an equation for the circulation is added to the original dynamical system and the detail of the reconnection process is carefully investigated [9].

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