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Energy fluxes in anisotropic turbulence

Naoto Yokoyama[1], Masanori Takaoka[2]
[1] Department of Mechanical Science and Bioengineering, Osaka University,
[2] Department of Mechanical Engineering, Doshisha University
e-mail: yokoyama@me.es.osaka-u.ac.jp

In anisotropic turbulence, weakly-nonlinear wave turbulence and strongly-nonlinear eddy turbulence usually coexist[1,2]. The inertial waves, the internal waves and the Alfvén waves are the wave components in the rotating turbulence, the stratified turbulence and the magnetohydrodynamic turbulence, respectively. The eddies consist of the Kolmogorov turbulence in all these three turbulent systems. The weak turbulence theory successfully gives the energy transfer in the weak-wave turbulence, and the Kolmogorov theory does in the eddy turbulence. The energy transfer in the transient wave-number range between the wave turbulence and the eddy turbulence is still under investigation[3].

Energy input by external force is redistributed via nonlinear interactions to wave numbers in the inertial subrange, and is finally dissipated by viscosity in Navier-Stokes turbulence. Because the energy is transferred not via one-to-one interactions but via a triad interaction and because a circular non-uniqueness exists in the triad interactions, it is not so easy to trace the energy transfer.

The energy flux plays a key role in energy-cascading process, and is defined as $P(k) = -\int_0^k T(k')dk'$ for a wave number in isotropic turbulence systems. The energy-transfer rate $T(k)$ is directly obtained in numerical simulations. The locality of the triad interactions is implicitly assumed in the definition. In the isotropic turbulence systems, both the energy flux and the energy-transfer rate are scalar-valued variables.

On the other hand, the energy flux in anisotropic turbulence systems such as rotating turbulence and stratified turbulence depends on the directions as well as the magnitudes of the wave numbers, and should be a geometric vector. Here, the local energy conservation in the wave-number space

$$T(k) + \nabla_{\boldsymbol{k}} \cdot \boldsymbol{P}(\boldsymbol{k}) = 0 \qquad (1)$$

is assumed to hold as is implicitly assumed in the isotropic turbulence. The energy-flux vector $\boldsymbol{P}(\boldsymbol{k})$ cannot be determined uniquely yet because the degree of freedom of the energy-flux vector is larger than the that of the energy-transfer rate.

In this work, we propose a way to uniquely determine the energy-flux vector in anisotropic turbulence by using the generalized (Moore-Penrose) inverse. The continuity equation of energy (1) is rewritten in a matrix form as $DP = -T$, where D is the fat matrix that corresponds to the divergence operator, and P and T respectively represent the generalized vectors consisting of all the wave-number components of $\boldsymbol{P}$ and T. Then, the vector P_* that has the minimal Euclidean norm among solution vectors is selected by using the generalized inverse matrix $D^+ = D^{\mathrm{T}}(DD^{\mathrm{T}})^{-1}$ as

$$P_* = -D^+T \qquad (2)$$

Because the null vector of the divergence operator is given by rotation of an arbitrary vector, the minimal-norm vector is irrotational. The selection of the minimal-norm vector corresponds to the assumption that the energy transfer is “efficient”.

Direct numerical simulations of stratified turbulence and rotating turbulence as well as isotropic turbulence are performed, and the energy-transfer rates are obtained in the simulations. The energy-flux vectors are obtained by using the generalized inverse in the three turbulence systems. In the stratified turbulence, the energy is mainly transferred to large vertical wave numbers via parametric subharmonic instability. The energy-flux vectors are directed to the large vertical wave numbers and consistent with the strong energy flux due to the parametric subharmonic instability[4,5]. Similarly, the energy-flux vectors in rotating turbulence are directed to large wave numbers normal to the rotation axis. It is consistent with the weak turbulence theory[6]. Moreover, the energy-flux vectors in the isotropic turbulence radiate outward as expected. It is found that the minimal-norm vector obtained using the generalized inverse well reproduces the direction of the energy flux in turbulence systems.

References

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