



## Hydrodynamic representation of the many-particle spin-1/2 Pauli equation for quantum plasmas

Pavel Andreev<sup>1</sup>

<sup>1</sup> Faculty of Physics, Lomonosov Moscow State University, Moscow, Russia  
andreevpa@physics.msu.ru (speaker):

The method of quantum hydrodynamics represents the dynamics of the many-particle quantum systems in terms of the collective variables. Choosing the concentration as the first collective variables we get a set of equations similar to the hydrodynamic equations. This feature gives name to the method. Collective effects have patterns in the three-dimensional physical space while the basic equations of quantum mechanics, the Schrodinger and Pauli equations, are written in the  $3N$ -dimensional configuration space, where  $N$  is the number of particles. Therefore, there is a task to represent the quantum mechanics in physical space. The quantum hydrodynamics method gives a natural projection of the configuration space to the physical space. The method is developed for the quantum plasmas and the neutral quantum gases with the short-range interaction.

Minimal coupling for the hydrodynamic spin effects reveals in eight-moment set of equations, where the standard five-moment approximation (concentration, momentum density and energy density) is supplemented with the spin density. Additional physical property leads to new phenomena, such as new waves and soliton solutions. These transverse waves appear with the frequency near the cyclotron frequency and near the cyclotron frequency multiplied by 0.001. Moreover, it provides existence of quantum vorticity and quantum helicity, which combines with classic plasma vorticity and helicity to get new general integrals of motion. Spin depended Alfvénic solitary structure is an example of new solitons.

Original way of description of the spin effects in the spin-1/2 quantum plasmas is the derivation of equations describing dynamics of all electrons simultaneously. Distinction from the classic plasmas is the force of spin-spin interaction in the Euler equation and the equation for the spin density evolution. Later development in this field demonstrated that independent description of electrons with the chosen spin projection leads to the separate-spin evolution quantum hydrodynamics. In this model the spin-up and spin-down electrons considered as two different fluids. The possibility of such description comes from the fundamental structure of the Pauli equation, where different spin projections are described by different equations traditionally combined in single equation via matrix notations.

The separate-spin evolution quantum hydrodynamic model reveals new phenomena. One of them is the spin-electron-acoustic wave. This way is related to the oscillation of electrons with different spin projection with different phases relatively positively charged ion background. Existence of this wave is related to difference of the partial Fermi pressures for the electrons with

different spin polarization. In the long-wavelength limit the spectrum of spin-electron-acoustic wave is linear. The phase velocity of the spin-electron-acoustic wave highly depends on the spin polarization of the electron gas. It appears as a longitudinal wave propagating parallel to the external magnetic field. Considering propagation of spin-electron-acoustic wave perpendicular to the external magnetic field find that the transverse electric field gives considerable contribution. Therefore, the extraordinary spin-electron-acoustic wave appears in this regime. Application of the separate spin evolution quantum hydrodynamics to study of the surface waves in plasmas demonstrates existence of the surface spin-electron-acoustic wave. Some other new wave and soliton solutions are found due to the application of the separate spin evolution quantum hydrodynamics. However, this model helped in solution of another fundamental problem.

The spin evolution equation contains the divergence of the collective spin current similar to the pressure in the Euler equation. An equation of state for the spin-current is required to get a complete model. However, papers on quantum plasma kept this term equal to zero due to the absence of a reasonable equation of state for degenerate electrons. While classical gas allows to use diffusive term providing some dissipation. Recently, an equation of state for degenerate electrons developed via the separate spin evolution quantum hydrodynamics. It appears to be proportional to difference of two terms, where each of these terms proportional to the partial concentration of electrons in fractional degree. Hence, it is not additive on partial concentrations. Further tracing of such effect via the kinetic model including separate spin evolution effects is made either. To this end, corresponding kinetic model called the separate spin evolution quantum kinetics is developed. This kinetic model includes the distribution function for each species of electrons (spin-up and spin-down). Moreover, the vector distribution function is introduced. The zeroth moment of the vector distribution function gives the hydrodynamic spin density. It provides meaning of the vector distribution function as the spin density in the six dimensional space. Application of the separate spin evolution quantum kinetics with the proper equilibrium distribution functions provides the account of effects related to the nontrivial contribution of the spin-current. It is similar to the fact that application of traditional kinetic equation includes temperature and pressure effect better than any equation of state for the pressure in the Euler hydrodynamic equation.