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Magnetic helicity and open magnetic fields

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1. Background

Magnetic helicity is a conserved quantity in ideal magnetohydrodynamics (MHD), and an approximately conserved quantity during reconnection [1]. It describes how magnetic field lines link within a volume.

The helicity H in a volume $\mathcal{V}$ is defined by

$$H = \int_{\mathcal{V}} \boldsymbol{A} \cdot \boldsymbol{B} \, \mathrm{d}\mathcal{V}, \quad (1)$$

where $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ is the magnetic field, and $\boldsymbol{A}$ is the vector potential. Under a gauge transformation $\boldsymbol{A} \rightarrow \boldsymbol{A} + \boldsymbol{\nabla}\psi$, the helicity transforms according to

$$H \rightarrow H + \int_{\partial\mathcal{V}} \psi \boldsymbol{B} \cdot \mathrm{d}\mathcal{S}. \quad (2)$$

Hence to ensure H is gauge invariant, the magnetic field must be closed, i.e. the normal component of the field must be zero on the boundary of the volume ($\boldsymbol{B} \cdot \widehat{\boldsymbol{n}}|_{\partial\mathcal{V}} = 0$).

For open magnetic fields, a gauge-invariant counterpart of H may be defined by considering the decomposition of the field $\boldsymbol{B} = \boldsymbol{B}_{\mathrm{p}} + \boldsymbol{B}_{\mathrm{c}}$ where $\boldsymbol{\nabla} \times \boldsymbol{B}_{\mathrm{p}} = 0$ and $\boldsymbol{B}_{\mathrm{p}} \cdot \widehat{\boldsymbol{n}}|_{\partial\mathcal{V}} = \boldsymbol{B} \cdot \widehat{\boldsymbol{n}}|_{\partial\mathcal{V}}$. In that case the relative helicity [2] is defined as:

$$H_{\mathrm{R}} = \int_{\mathcal{V}} = (\boldsymbol{A} + \boldsymbol{A}_{\mathrm{p}}) \cdot (\boldsymbol{B} - \boldsymbol{B}_{\mathrm{p}}) \, \mathrm{d}\mathcal{V}, \quad (3)$$

where $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ and $\boldsymbol{B}_{\mathrm{p}} = \boldsymbol{\nabla} \times \boldsymbol{A}_{\mathrm{p}}$. This quantity is independent of the choice of gauge.

2. Solar application

Magnetic helicity enters the corona due to the emergence and twisting of magnetic flux at the photosphere. Helicity injection in a given solar hemisphere is predominantly of one sign, and then it is approximately conserved, so the helicity is expected to continuously build up in the corona, until it is expelled in coronal mass ejections [5].

There is considerable interest in modeling magnetic helicity in the corona. The field is open because it crosses the photosphere, and extends out into the solar wind. Two approaches to estimate the helicity are 1. to construct models for the coronal field as a nonlinear force-free field using photospheric boundary conditions on the field, or via MHD simulations incorporating data, and to use eq. (3), or 2. to calculate the relative helicity flux through the photospheric boundary, based on the time evolution of the magnetic field and velocity field on the boundary [4]. A test of 1. using magnetic fields known in a finite volume showed that exisiting techniques can reliably estimate magnetic helicity for a range of test fields [7]. However, it remains unclear whether modeling methods allow accurate determination of the helicity of magnetic fields on the Sun [3, 6].

The calculation of relative magnetic helicity is based on the current-free magnetic field ($\boldsymbol{B}_{\mathrm{p}}$), which is constructed using Neumann boundary conditions on all surfaces of $\mathcal{V}$. Here we consider a cartesian volume, with the lower boundary representing the solar photosphere. From the Thomson theorem, we find that a current-free field with Neumann boundary condition on the bottom and top, and periodic lateral boundaries will have lower magnetic energy than the current-free field with Neumann boundary conditions on all six boundaries. In the former case, the condition $\boldsymbol{B}_{\mathrm{p}} \cdot \widehat{\boldsymbol{n}}|_{\partial\mathcal{V}} = \boldsymbol{B} \cdot \widehat{\boldsymbol{n}}|_{\partial\mathcal{V}}$ does not hold on the lateral boundaries. However, use of this potential field results in the relative helicity (3) being gauge-dependent. To make use of the periodic current-free field, we need to modify the original definition of the relative magnetic helicity to ensure gauge-independency.

In this talk, we will demonstrate methods for determining the helicity of open magnetic fields and present a new definition for the relative magnetic helicity based on the periodic potential field. The measures of helicity will be applied to known force-free magnetic fields, and to nonlinear force-free models for solar coronal magnetic fields.

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