3rd Asia-Pacific Conference on Plasma Physics, 4-8,11.2019, Hefei, China

Deterministic representation of chaotic attractors and capture of all homoclinic points in Henon map

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The Henon map [1] is a model that exhibits the same property as the Lorenz system. It was developed to describe the atmospheric turbulence on the basis of the Navier-Stokes equation. Since then, numerous researchers have used this map as the simplest model to describe the chaotic behavior in various dissipative systems. The strange attractor, which appears as the unstable manifold of the Henon map, is regarded as a typical model of unpredictable motion, and it is well-known that its trajectory is nondeterministic. In the current study, we present an analytic function that describes the unstable manifold, including the stable manifold, of the Henon map. We use the Borel-Laplace transform and asymptotic expansions to construct this function [2-4]. It becomes a new special function for describing the global solution to a nonlinear equation. We show the stable and unstable manifolds of the Henon map depicted by use of the function in Figure 1.

The Henon map belongs to the non-integrable systems. The fact that a dynamical system is non-integrable is identical with that the system has one (transverse) homoclinic point. This was first proved by Poincare and he found a homoclinic point in his study for the three-body problem [5]. Generally, to find homoclinic points, even though it is only one point, is a difficult task; however, if one succeeds to identify all homoclinic points, it indicates that every event extending from the past to future can determine in a dynamical system. If one can calculate all homoclinic points concretely, the capture of chaos and future prediction become possible.

Several researchers including Poincare have been trying to find homoclinic points; however, there exists no study that could detect more than one homoclinic point so far. Gelfreich and Sauzin found a homoclinic point in the Henon map for the case that the system becomes elliptic (i.e., Hamiltonian system) and calculated the homoclinic point in a high accuracy using the resurgent analysis [6]. In the current study, we present a new method for calculating all homoclinic points in the Henon map using the function described above and the Newton's method. In Figure 2. we show the homoclinic points of the Henon map calculated by use of the analytic function constructed here. In the figure, 211772 homoclinic points are depicted. The set of homoclinic points is a structure realized in $\mathbf{C}^2$ (complex 2D). We depict one of the 3D ($\mathbf{R}^3$) cross-sections of the structure in the figure. Although there exists a round-off error, the obtained result is mathematically rigorous and no approximation is required.

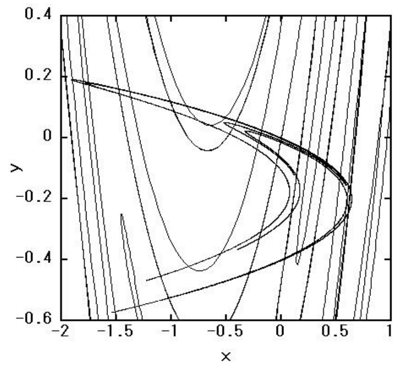

Figure 1

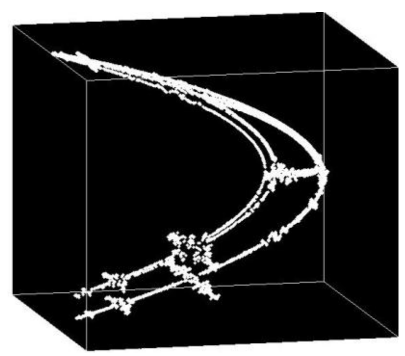

Figure 2

References

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