3rd Asia-Pacific Conference on Plasma Physics, 4-8, 11.2019, Hefei, China

MHD equilibria via simulated annealing and their stability – negative energy modes and additional constraints

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There exists a variety of dynamics that are well described as Hamiltonian systems. An ideal fluid dynamics, including magnetohydrodynamics (MHD), is one of the Hamiltonian systems. For the description of such fluid dynamics, we often adopt noncanonical variables [1]. Then the corresponding Poisson bracket can have kernels, called Casimirs, that foliate the phase space. A set of phase points that have same values of the Casimirs is a Casimir leaf. Therefore, the system evolves along a contour of the Hamiltonian on the Casimir leaf.

Let us write the evolution equation of a vector of phase space variables u as $\partial u/\partial t = \mathcal{J}(\delta F[u]/\delta u)$, where $\mathcal{J}$ expresses the Poisson bracket, $F[u] := H[u] + \sum_i C_i[u]$ is the energy-Casimir functional with $H[u]$ the Hamiltonian functional and C_i the Casimirs, and $\delta F[u]/\delta u$ is the functional derivative of $F[u]$. A stationary state is given by $\delta F[u]/\delta u = 0$, which is an extremum or a stationary point of the Hamiltonian on the Casimir leaf [2, 3].

Utilizing this fact, a method for calculating a stationary vortex states of two-dimensional fluids was developed, where an artificial advection changes the energy of the system monotonically while preserves the Casimirs, leading to a stationary state [4]. This idea was shown to be applicable to any Hamiltonian systems [5], and various numerical examples were shown together with discussions on the stability of the stationary states [6]. The artificial advection method was further extended to impose additional constraints using the Dirac bracket, and for example to effect smoothing by introducing a metric operator in their symmetric bracket [7]. The extended method is named simulated annealing (SA). A variety of numerical examples were also presented [7].

We have been extending the application of the SA to MHD equilibrium calculations. We have first succeeded to obtain equilibria of the low-beta reduced MHD in a doubly-periodic rectangular domain [8]. Since the SA dynamics is restricted on a Casimir leaf, we may need to put the initial condition on a desired Casimir leaf. Therefore we have also developed a method to pre-adjust the initial condition [9]. We have applied the SA also for a cylindrical plasma, and have obtained equilibria with magnetic islands [10]. Moreover, we have obtained toroidal axisymmetric equilibria, large-aspect-ratio and circular cross-section tokamaks and toroidally-averaged stellarators [11], by using high-beta reduced MHD.

If we construct the SA so that the energy decreases monotonically, an obtained equilibrium has a minimum energy. However, as mentioned above, an equilibrium can generally have maximum, minimum or stationary energy on the Casimir leaf. Therefore the SA cannot achieve all types of equilibria by the present construction. Especially, the SA cannot obtain the stationary (saddle) energy equilibrium by either decreasing or increasing energy of the system. We may need to impose additional constraints that restrict the artificial dynamics of the SA on a specific “curve” on the original Casimir leaf. In order to understand the stability nature of equilibria and to develop a method to impose the additional constraints, we have examined dynamics of a heavy top.

A heavy top has a six-dimensional phase space and two Casimir invariants by using noncanonical variables. We have obtained reasonable understanding in comparisons among linear stability of stationary states, eigenvalues of the Hessian matrix of the energy-Casimir function, mode energy including negative one, as well as linear stability of the SA dynamics. We have also constructed a Dirac bracket that keeps one of the variables unchanged, and succeeded to obtain a stationary state as an energy minimum on the Casimir leaf with the additional constraint. The obtained state is not a stationary state originally. Therefore this method can be applicable to restrict the SA dynamics on the original Casimir leaf. We have formulated the Dirac bracket for the low-beta reduced MHD by the same way as the heavy top.

Acknowledgement

The author is grateful to Prof. P. J. Morrison for fruitful discussions. This work was supported by Japan / U. S. Cooperation in Fusion Research and Development.

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