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Three dimensional azimuthal magnetorotational instability of a MHD flow

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Since the rediscovery of Velikhov's and Chandrasekhar's [1,2] results by Balbus and Hawley [3], the magnetorotational instability (MRI) has aroused strong interest as a promising mechanism for triggering turbulence in the flow of an accretion disk and for promoting outward transport of angular momentum, while the matter accretes to the center. For an accretion disk, the Keplerian flow, a cylindrically symmetric flow with the profile of rotational velocity $U_\theta \propto r^{-1/2}$, satisfies the force balance $U_\theta^2/r = \Omega^2(r)r = -\nabla\Phi; \Phi \propto 1/r$. Here r is the distance from the axis of symmetry. The combined effect of fluid rotation and the imposed axial magnetic field makes unstable the Keplerian flow of an ideal conducting fluid, for which the fluid viscosity and the electric resistivity are neglected.

We consider a rotating flow $\boldsymbol{U} = r\Omega(r)\boldsymbol{e}_\theta$, and a helical magnetic field $\boldsymbol{B} = r\mu(r)\boldsymbol{e}_\theta + B_z\boldsymbol{e}_z$, where $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_z$ are the unit vectors in the azimuthal and axial directions. We deal primarily with the azimuthal magnetic field $B = r\mu(r)\boldsymbol{e}_\theta$ and study the azimuthal MRI (AMRI) in three dimensions locally by the Wentzel-Kramers-Brillouin (WKB) method, valid for short-wavelengths, and globally, from the Hamiltonian viewpoint, by calculating the wave energy.

For the WKB approximation, the radial wavelength $2\pi/q$ is assumed to be much shorter than the radial characteristic length L, where q is the wavenumber in the radial direction. Traditionally, the WKB method has been applied to the MHD equations, which has a pitfall of overlooking some terms relevant to the non -axisymmetric disturbances [4]. A more careful approach is to deduce a differential equation for a single component of the Lagrangian displacement field, and then apply the WKB approximation.

In ideal MHD, we make the WKB approximation to the Hain–Lüst equation for the radial Lagrangian displacements [5, 4]. For the Keplerian flow with current-free magnetic field $B_\theta \propto 1/r$, the maximum growth rate is the Oort A-value 0.75, which is reached at a finite magnetic field when $k \to \infty$ and $m \to \infty$, where k and m are the axial and azimuthal wavenumber respectively. But for the $k \to 0$ mode, maximum growth rate increases, beyond the Oort A-value and is linear in the magnetic strength.

For experiments with liquid metals, the effects of both the viscosity ν and the magnetic diffusivity η cannot be ignored. Also because of the low electric conductivity, the magnetic Prandtl number $Pm = \nu/\eta$ is very small and the case of $Pm = 0$ is referred to as the inductionless limit [6]. We extend the Hain–Lüst equation by incorporating the effect of ν and η and apply the WKB method to it. Defining the magnetic Rossby number as $Rb = \mu'(r)/2\mu$, the short axial wavelength mode $k \to \infty$ is found by the traditional WKB approximation [6] but the long axial-wavelength mode $k \to 0$ is new. For the Keplerian flow, $k \to \infty$ mode is excitable for $Rb > 25/32$ and the $k \to 0$ mode is excitable for $Rb < -1/4$. The later makes the Keplerian flow in current-free magnetic field $(Rb = -1)$, unstable.

The ideal MHD is a Hamiltonian system and the magnetic field lines are frozen into the fluid, i.e. the magnetic flux is conserved

$$\frac{D}{Dt}\int_S \boldsymbol{b}\cdot d\boldsymbol{S} = 0,$$

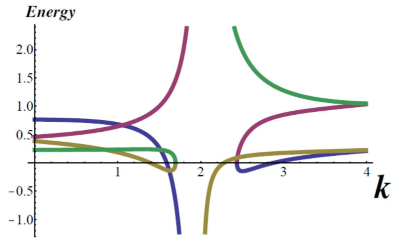
where $\boldsymbol{b}$ is the magnetic field and S is an arbitrary material surface. Lagrangian displacement $\boldsymbol{\xi}$ describes a perturbation of the trajectories of a fluid particle. Assume that the disturbance is kinematically accessible i.e. that the magnetic flux remains unchanged. This sort of disturbance is called the isomagnetovortical [7], for which the first-order disturbances of the vorticity and magnetic field $\boldsymbol{\omega}_1$ and $\boldsymbol{b}_1$ take the form

$$\boldsymbol{\omega}_1 = \boldsymbol{\xi}\times\boldsymbol{\omega}_0 + \boldsymbol{\eta}\times\boldsymbol{B},$$
$$\boldsymbol{b}_1 = \nabla\times(\boldsymbol{\xi}\times\boldsymbol{B}),$$

where $\boldsymbol{\omega}_0$ and $\boldsymbol{B}$ are the basic steady vorticity and magnetic field, a state in equilibrium, and $\boldsymbol{\eta}$ is a supplementary displacement field. We manipulate the wave energy of the isomagnetovortical disturbance as

$$E = \int \boldsymbol{\omega}_0\cdot\left(\frac{\partial\boldsymbol{\xi}}{\partial t}\times\boldsymbol{\xi}\right) + \boldsymbol{B}\cdot\left(\frac{\partial\boldsymbol{\xi}}{\partial t}\times\boldsymbol{\eta} - \boldsymbol{\xi}\times\frac{\partial\boldsymbol{\eta}}{\partial t}\right).$$

For a rigid rotation in the magnetic field with $m = -5$, $\Omega = 0.25$, $\mu = 1, m = 1$ and $B_z = 0$, we calculated the wave energy as in the following figure.



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