



Variational integration for ideal MHD and formation of current singularities

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Coronal heating has been a long-standing problem in solar physics. Parker's conjecture [1] that the current density in the corona is distributed in the form of singular current sheets that produce the required heating has been controversial. In ideal MHD, can genuine current singularities emerge from a smooth 3D line-tied magnetic field? To resolve this issue computationally, the numerical scheme must preserve magnetic topology exactly to avoid artificial reconnection. We develop a novel variational integrator for ideal MHD by discretizing Newcomb's ideal MHD Lagrangian on a moving mesh using discrete exterior calculus [2]. The motion of the mesh advects the discrete magnetic flux exactly, such that the scheme is free of artificial reconnection. With this method, we confirm that the nonlinear solution to the ideal Hahn—Kulsrud—Taylor (HKT) problem in 2D yields a singular current sheet [3], agreeing well with an analytical solution we obtain [4] using the boundary layer approach developed by Rosenbluth et al. for the $m=1$ internal kink problem [5]. We then extend the ideal HKT problem to 3D line-tied geometry [6,7], the effect of which is crucial in the controversy over Parker's conjecture. The linear solution, which is singular in 2D, is found to be smooth. However, with finite amplitude, it can become pathological when the system is sufficiently long. The nonlinear solution turns out to be smooth for short systems. Nonetheless, the scaling of peak current density vs. system length suggests that the nonlinear solution may become singular at a finite length. Albeit not yet a conclusive resolution, our results contribute to our understanding of the Parker problem by exemplifying how the line-tied geometry influences the formation of current singularities.

References

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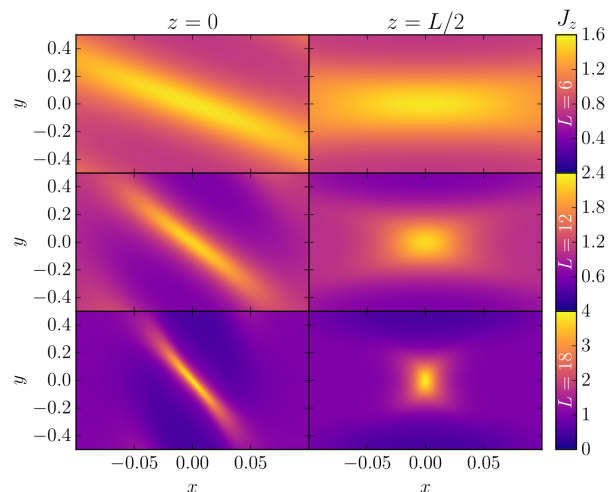


Figure 1. The distribution of the current density J_z at the footpoint ($z=0$, left) and the mid-plane ($z=L/2$, right) becomes more concentrated as the system length L increases (from top to bottom).