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## **Action principles for relativistic extended magnetohydrodynamics: A unified theory of magnetofluid models**

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In a fluid dynamical framework, non-dissipative models are expected to be derived from action principles (APs) and associated Hamiltonian formalism. Derivation through AP brings a strong credibility to the model. The key to developing APs for fluid models is the implementation of various constraints, such as particle number and entropy conservation. There are multiple formalisms depending on how the constraints are implemented — first, Lagrangian description AP where the constraints are attached to fluid elements' “attributes” [1], second, constrained least AP in which the constraints are embedded into an action with Lagrange multipliers [2,3], and third, bracket AP where the constraints are implemented via the degeneracy of a Poisson bracket [4,5]. The previously proposed Hamiltonian formalisms for the various magnetohydrodynamic models can be categorized into one of these APs.

In this work, we developed the latter two APs (i.e., the constrained least AP and the bracket AP) for relativistic extended MHD [6], which includes Hall effect and electron inertia effect as well as relativistic effects [7]. This AP can be reduced to the Hamiltonian formalism of other MHD type models (e.g., ideal MHD, Hall MHD, and inertial MHD). We also found that the constrained least AP and the bracket AP are connected via variable transformation from the Clebsch potential to the physical field variables.

### References

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