



## The use of Bayesian inference to model complex systems

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The use of Bayesian inference is flourishing among diverse fields, ranging from astronomy to climate science. The Bayesian approach to inference in fusion plasmas, developed by multiple authors<sup>1,2,3,4</sup>, involves the specification of an initial prior probability distribution function,  $P(I)$ , for the vector of plasma parameters,  $I$ , which is then updated by taking into account information that the measurements provide through the likelihood probability distribution  $P(D|I)$ , where  $D$  is the measurement vector, and the notation  $D|I$  denotes a forward diagnostic model describing the response data  $D$  to plasma parameters  $I$ . The result is the posterior distribution  $P(I|D)$ , the conditional probability assigned after the relevant evidence  $D$  is taken into account, given by Bayes' formula  $P(I|D) = P(D|I) P(I)/P(D)$ .

CI Hole and collaborators have developed a model validation framework based on Bayesian probabilistic methods to distinguish between competing equilibrium theories. Utilising this Bayesian framework, implemented in the code suite Minerva<sup>5</sup>, we have: demonstrated current tomography using motional Stark effect observations, and data from magnetic pick up coils and flux loops<sup>6,7</sup>; developed a new diagnostic technique to identify and remove outlier observations associated with diagnostics falling out of calibration or suffering from an unidentified malfunction<sup>8</sup>; developed models for Thomson Scattering<sup>9</sup>; implemented nested-sampling<sup>10</sup>, and optimised the posterior distribution<sup>11</sup>. We have also computed an estimate of the energetic particle pressure using a generalised force balance equation as a constraint<sup>12</sup>. The latter produces an energetic pressure profile whose core pressure is 30% of the thermal pressure - commensurate with other beam discharge plasmas with similar heating.

In this topical review the use of Bayesian inference in plasma physics, it's growing use in the inference of fast ions, and identify how these developments can be ported to other fields of research. An illustration is a spin-off project funded by the Australian Signals Directorate to develop Bayesian inference of global computer network dynamics using limited sensor data.

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