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Methodology for extracting and modeling electron-scale effects in multi-scale plasma turbulence

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Direct numerical simulations (DNS) of plasma turbulence with utilizing massively parallelized supercomputers promote the understanding of multi-scale nature of plasma turbulence. Recent multi-scale plasma turbulence researches revealed the existence of cross-scale interactions between short-wavelength electron-scale turbulence and long-wavelength ion-scale turbulence in magnetically conned plasmas [1,2,3]. Although more case studies are demanded for clarifying the condition of cross-scale interactions, the number of analyses is limited so far because of its extremely high computational cost. Therefore, in anticipation of understanding the generic features and of constructing a reduced simulation model of multi-scale turbulence, we discuss the methodology for extracting and modeling the contributions from small-scale fluctuations on large-scale fluctuations.

Mori-Zwanzig projection operator method [4], which has been developed in non-equilibrium statistical physics researches, describes the governing equation $f(t)$ of variables of interests $u(t)$ in the form of generalized Langevin equation,

$$f(t) = \Omega \cdot u(t) - \int_0^t \Gamma(s) \cdot u(t-s)ds + r(t), \qquad (1)$$

where using a statistical average $\langle \cdots \rangle$,

$$\Omega = \langle fu^* \rangle \cdot \langle uu^* \rangle^{-1}, \qquad (2)$$

$$\langle r(t)\frac{du^*}{dt} \rangle = \Gamma(t) \cdot \langle uu^* \rangle, \qquad (3)$$

$$\langle r(t)u^* \rangle = 0. \qquad (4)$$

In other words, in equation for variables of interests, contributions from the other variables are extracted as the memory function $\Gamma(s)$ and the noise term $r(t)$. Applying the projection operator method for dividing the system into large-scale fluctuations (= variables of interests) and small-scale fluctuations (= the other variables), one expects to extract and model the small-scale contributions on large-scale fluctuations.

As an example, we examined the applicability of the Mori-Zwanzig projection operator method to the Kuramoto-Sivashinsky equation,

$$\frac{\partial u(x,t)}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} = 0, \qquad (5)$$

which is one of examples of non-equilibrium open systems of one-dimensional turbulence. We evaluated the memory functions and the noise term from the DNS data of Kuramoto-Sivasinsky turbulence. Then we confirmed that the generalized fluctuation-dissipation theorem of the second kind, Eq. (3), which states the relation between the memory function and the time-correlation of the noise term, was numerically satisfied, as shown in Figure. Additionally, using the memory function and the noise term obtained from DNS data, we constructed a reduced model for large-scale fluctuations including small-scale contributions. It was demonstrated that such a sub-grid-scale modeling reproduces the energy spectrum same as the DNS of Kuramoto-Sivasinsky turbulence.

References
[1] S. Maeyama, et al., Phys. Rev. Lett. 114, 255002 (2015).
[2] N. T. Howard, et al., Nucl. Fusion 56, 014004 (2016).
[3] S. Maeyama, et al., Phys. Rev. Lett. 119, 195002 (2017).
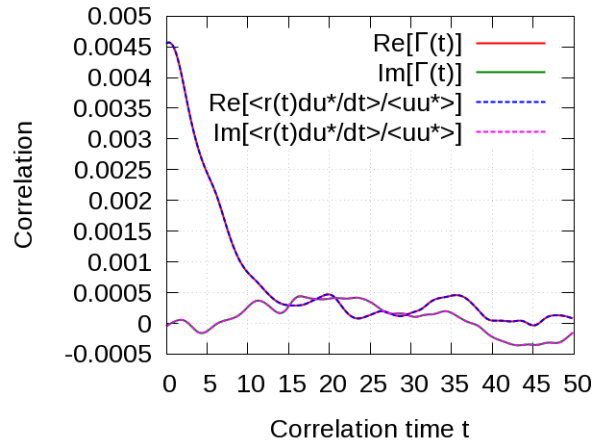[4] H. Mori, Prog. Theor. Phys. 33, 423 (1965).

Figure. Memory function $\Gamma(s)$ and temporal correlation of the noise term $r(t)$, evaluated from the DNS data of Kuramoto-Sivasinsky turbulence.