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Free-boundary stability analysis of tokamak plasmas by ERMHDT (Eigenvalue code for Resistive MHD in Toroidal geometry)

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We have developed an eigenvalue code ERMHDT for resistive magnetohydrodynamics (MHD) linear stability analysis of tokamak plasmas including toroidal rotations. The code has been recently upgraded for free-boundary calculations. We have achieved good agreements of ideal MHD growth rates with other existing codes for Solovév equilibria[1]. Also we have successfully captured that free-boundary ideal kink modes and fixed-boundary tearing modes are smoothly connected by the free-boundary resistive MHD stability analysis.

In the plasma region, the ERMHDT code [2] solves linearized MHD equations including resistivity as well as equilibrium toroidal plasma rotation. The Ohm's law is solved for the vector potential, instead of solving the induction equation for the magnetic field. The linearized equations are Fourier transformed in time, and thus we solve an eigenvalue problem under appropriate boundary conditions.

In the vacuum region, we solve the Laplace equation for a magnetic scalar potential χ. The perturbed magnetic field in the vacuum region is expressed as $\tilde{\boldsymbol{B}} = \nabla\chi$. This automatically satisfies $\nabla \times \boldsymbol{B} = \boldsymbol{0}$. From $\nabla \cdot \boldsymbol{B} = 0$, we obtain the Laplace equation $\nabla^2\chi = 0$.

We impose boundary conditions at the plasma–vacuum interface and at the metallic wall surrounding the vacuum region. At the plasma–vacuum interface, we impose continuity of total pressure as well as normal component of the perturbed magnetic field. We assume there is no equilibrium surface current at the interface. At the metallic wall, we assume that the wall is a perfect conductor and set the perturbed magnetic field normal to the wall to be zero.

We solve the eigenvalue problem in a weak form. The weak form is discretized by the Fourier series expansions in poloidal and toroidal directions, and the finite element method in a minor radius direction. These procedures are same as the CASTOR code[3]. The discretized eigenvalue problem is solved either by an inverse power iteration with an eigenvalue shift, or by the ARPACK library[4].

The ERMHDT code has a unique feature: we can use a different basis function for each variable in the finite element method. It is known that the radial component of perturbed velocity field should be expressed by a basis one order higher than the other two components for avoiding the spectral pollution [5]. Our code can examine a nice combinations of the basis functions in the view point of convergence properties as well as magnitudes of the discretization error.

First, we compared growth rates of ideal MHD modes for Solovév equilibria with other codes, and have obtained good agreements. A number of codes were compared in [3] for both fixed-boundary and free-boundary modes, and we compared our results with them. We have obtained almost identical growth rates for all cases tested.

Second, we have successfully captured continuous transition between free-boundary ideal kink modes and fixed-boundary tearing modes for large-aspect-ratio, circular-cross-section and zero-beta tokamak equilibria [6]. Figure shows the normalized growth rate for the toroidal mode number $n = 1$. The aspect ratio is 10. In the horizontal axis label, q_a denotes the safety factor at the plasma edge. The normalized resistivity (the inverse of the Lundquist number) is denoted by η. The growth rate for the free-boundary ideal kink mode shows the behavior well-known in cylindrical plasmas. On the other hand, a fixed-boundary tearing mode becomes unstable if a corresponding rational surface exists inside the plasma. By the free-boundary calculations including resistivity, these two kinds of modes are smoothly connected.

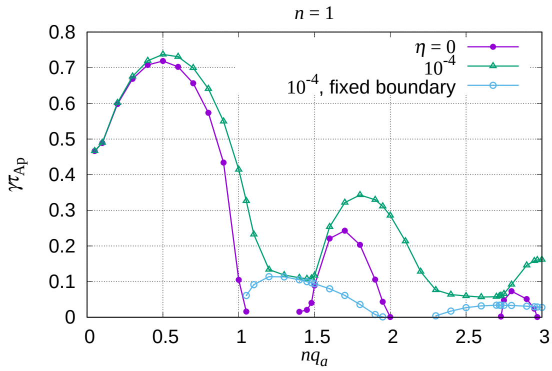

Figure 1: The normalized growth rates are plotted versus nq_a for the Shafranov equilibrium, where n is the toroidal mode number and q_a is the edge safety factor. Free-boundary ideal ($\eta = 0$) kink mode and fixed-boundary tearing mode are connected smoothly.

References

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