

## Anomalous transport and acceleration of cosmic rays in the presence of MHD turbulence

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The motion of cosmic rays (energetic particles) in the MHD turbulence can be quite complex. They can at times be trapped by a large amplitude wave packet and stay there for a long time, or can they make almost ballistic motion without much influenced by the turbulence field. Diffusion of the cosmic rays can thus be normal (Gaussian), sub-, or super-diffusive, depending on turbulence properties (energy, spectrum, presence of intermittency, etc) and also on the time scale considered.

We have been taking multiple approaches to tackle this problem. Here, we will be introducing some of our efforts as below:

(1) Test particle simulation: In a given finite amplitude MHD turbulence, one can compute trajectories of the cosmic rays and evaluate their diffusion properties. Figure 1 gives an example of "purely 2-d" cross-field diffusion, in which all the magnetic field lines are assumed to be parallel everywhere (so that the cross-field diffusion due to "braided" field lines is artificially suppressed) [1,2]. The top panel shows some typical time series of the cosmic rays. Apparently, it is distinct from usual Brownian motion, in that it includes many segments of "halts", corresponding to the particle being trapped by magnetic islands. The bottom panel is the diffusion coefficient plotted versus time scale. As expected, the diffusion is sub-diffusive for a certain range of the time-scale. At larger time scales, the diffusion can be regarded normal.

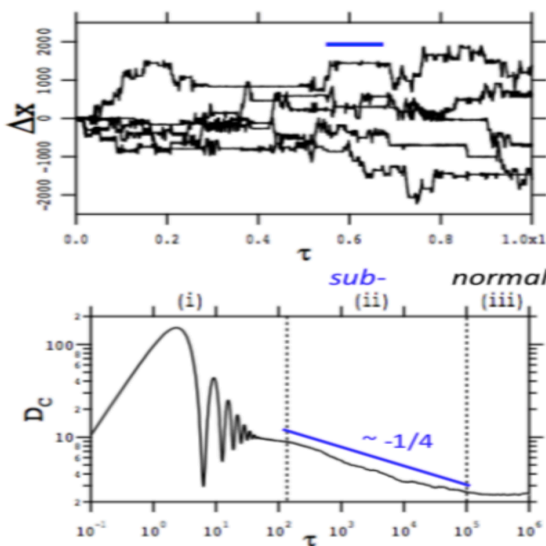


Fig.1. (above) Typical motion of cosmic rays. (bottom) Time scale dependent diffusion coefficient.

(2) Walk-stick model: Weeks et al [3] proposed a simple model to generate a random walk sequence that can exhibit both sub- and super-diffusive characters by

letting the random walker alternate between flights (steps of constant velocity) and sticking (pauses between flights). Flight and sticking time probability distribution functions are specified by a Levy distribution with different indices. The cosmic ray trajectories generated by this walk-stick model are applied to the diffusive shock acceleration process [4].

(3) Fractional convection-diffusion model: A natural model to describe ensemble of particles that undergo anomalous transport processes is the fractional diffusion equation, in which the time and/or spatial derivative may involve fractional differentiation operators [5]. First, we briefly introduce the concept of the fractional differentiation/integration operators and explain how to evaluate them numerically. As an important application of this model, we consider the diffusive shock acceleration process by solving numerically the fractional convection diffusion equation:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = \kappa \frac{\partial^\alpha f}{\partial x^\alpha} + \delta(x)H(t) \quad (*)$$

where the r.h.s. involves the fractional differentiation of the order  $\alpha$ . The results will be compared with those obtained by test particle simulations using sub- and super-diffusive particles.

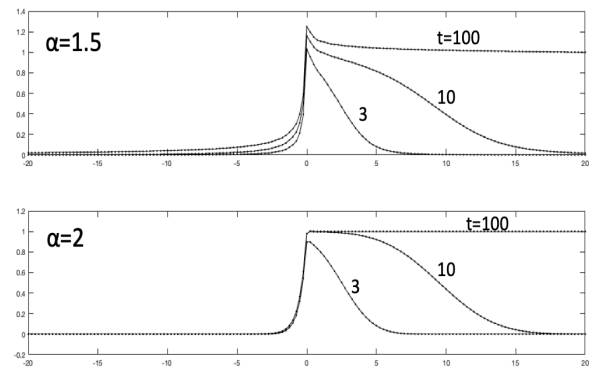


Fig.2. Numerical solution of (\*) with  $\alpha = 1.5$  (super-diffusive) and  $\alpha = 2$  (normal).

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