The motion of cosmic rays (energetic particles) in the MHD turbulence can be quite complex. They can at times be trapped by a large amplitude wave packet and stay there for a long time, or can make almost ballistic motion without much influenced by the turbulence field. Diffusion of the cosmic rays can thus be normal (Gaussian), sub-, or super-diffusive, depending on turbulence properties (energy spectrum, presence of intermittency, etc) and also on the time scale considered.

We have been taking multiple approaches to tackle this problem. Here, we will be introducing some of our efforts as below:

(1) Test particle simulation: In a given finite amplitude MHD turbulence, one can compute trajectories of the cosmic rays and evaluate their diffusion properties. Figure 1 gives an example of "purely 2-d" cross-field diffusion, in which all the magnetic field lines are assumed to be parallel everywhere (so that the cross-field diffusion due to "braided" field lines is artificially suppressed) [1,2]. The top panel shows some typical time series of the cosmic rays. Apparently, it is distinct from usual Brownian motion, in that it includes many segments of "halts", corresponding to the particle being trapped by magnetic islands. The bottom panel is the diffusion coefficient plotted versus time scale. As expected, the diffusion is sub-diffusive for a certain range of the time-scale. At larger time scales, the diffusion can be regarded normal.

(2) Walk-stick model: Weeks et al [3] proposed a simple model to generate a random walk sequence that can exhibit both sub- and super-diffusive characters by letting the random walker alternate between flights (steps of constant velocity) and sticking (pauses between flights). Flight and sticking time probability distributions of the cosmic rays are evaluated by test particle simulations using sub- and super-diffusive particles.

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = \kappa \frac{\partial^\alpha f}{\partial x^{\alpha}} + \delta(x)H(t)$$

where the r.h.s. involves the fractional differentiation of the order $\alpha$. The results will be compared with those obtained by test particle simulations using sub- and super-diffusive particles.

Fig. 2. Numerical solution of (*) with $\alpha = 1.5$ (super-diffusive) and $\alpha = 2$ (normal).

References: