



## Relation of the Vlasovian and of the N-body descriptions of microscopic plasma physics

D. F. Escande

<sup>1</sup> Aix-Marseille Université, CNRS, PIIM, UMR 7345, Marseille, France

e-mail : dominique.escande@univ-amu.fr

Until recently, the mechanical N-body description of microscopic plasma physics was deemed impossible, and was substituted with the Vlasovian description, which brought a huge amount of theoretical knowledge in this physics. However, in the last years the N-body description of microscopic plasma physics turned out to be manageable, in particular for Landau damping [1,2,3], for Debye shielding [1,2,3], and for Coulomb scattering [4]. Its application to the first two cases revealed two important features of the *Vlasovian limit: it is singular and it corresponds to a renormalized description of the actual N-body dynamics.*

The N-body description of Landau damping and Debye shielding uses a spatially uniform granular distribution, which is the analog of  $f_0(\mathbf{v})$  in a Vlasovian setting. This granular distribution is made up of monokinetic beams, and each beam is a cubic array of particles. It is called multi-beam-multi-array.

The singularity of the Vlasovian limit shows up in various ways: (i) The dielectric function of a multi-beam-multi-array does not converge toward the Vlasovian one, when the density of the beam velocities increases, because neither zeros nor poles match. (ii) As shown by the echo experiment, the van Kampen/beam modes related to a given Langmuir wave are actually present in a granular plasma. In the N-body approach, the phase mixing of such modes plays an essential role for the time reversibility of the amplitude of an unstable wave too, while it is absent in the Vlasovian calculation. (iii) In this approach, the term of initial conditions in Landau's calculation of Langmuir waves is the sum of the ballistic potentials of the N electrons. When the density of the beam velocities increases, such a sum has no limit, which shows Landau's term of initial conditions to be a singular limit, and explains while the latter has no physical interpretation. (iv) Adding a test particle to the N-body system does not provide the shielded potential of this particle, in contrast with the Vlasovian case, (v) In the N-body approach, Landau damping occurs without requiring the distribution function to be analytically continuable, like in the usual Vlasovian approach, which is better for the experimental observability of the phenomenon.

One of the simplest examples of a renormalized potential is the Debye shielded potential. In the N-body approach, the Debye shielded potential of a particle is a mean-field potential produced by its Coulomb deflections of all other particles, which makes explicit what is traditionally called "dressed particle". Both its mean-field and BBGKY derivations show that Vlasov equation deals with a mean-field potential. Therefore, the Vlasovian dielectric function is a renormalized version of that of a multi-beam

multi-array, and a Vlasovian Langmuir wave is the renormalized version of a set of beam modes of the N-body system. The renormalized dielectric function enables the calculation of the shielded potentials of the N particles of the granular plasma considered here, or of a test particle added to a Vlasovian plasma.

We now sketch the principle of the three derivations of Landau damping using the N-body approach: (i) A pedestrian, short, yet rigorous, one germane to Kaufman's in a Vlasovian setting [5]. (ii) One showing waves damp because of phase mixing à la van Kampen. (iii) One showing that the Vlasovian limit is singular and corresponds to a renormalized description of the actual N-body dynamics. These derivations are accessible to students who know Newton's second law and elementary calculus. They use neither statistical settings, nor partial differential equations. The first one uses neither Laplace transform, nor contour integration, nor analytic continuation. All three derivations use the one Component Plasma model, which considers the plasma as infinite, with spatial periodicity L in three orthogonal directions, made up of N electrons in each elementary cube with volume  $L^3$ , and with a uniform ionic neutralizing background.

The calculations are similar to Vlasovian ones. They use the linearization of N-body dynamics with Coulomb interactions, and the perturbation from ballistic orbits of electrons of a multi-beam-multi-array. At the end of the calculation, one substitutes the discrete sums over particles with integrals over a smooth distribution function  $f_0(\mathbf{v})$ . As done in the derivations of Vlasov equation from N-body dynamics, Coulomb potential is smoothed at short distances, which reduces partially the strength of collisions.

### References

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