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Singular measures and information capacity of turbulent cascades

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How weak is the weak turbulence? Here we analyze turbulence of weakly interacting waves using the tools of information theory. It offers a unique perspective for comparing thermal equilibrium and turbulence. The mutual information between modes is stationary and small in thermal equilibrium, yet is shown here to grow with time for weak turbulence in a finite box. We trace this growth to the concentration of probability on the resonance surfaces, which can go all the way to a singular measure. The surprising conclusion is that no matter how small is the nonlinearity and how close to Gaussian is the statistics of any single amplitude, a stationary phase-space measure is far from Gaussian, as manifested by a large relative entropy. This is a rare piece of good news for turbulence modeling: the resolved scales carry significant information about the unresolved scales. The mutual information between large and small scales is the information capacity of turbulent cascade, setting the limit on the representation of subgrid scales in turbulence modeling.

There are two quite different perspectives to look at the evolution of a statistical system: fluid mechanics and information theory.

The first one is the continuum viewpoint, where a Hamiltonian evolution of an ensemble is treated as an incompressible flow in a phase space. Such flows generally mix leading to a uniform microcanonical equilibrium distribution. On the contrary, to deviate a system from equilibrium, one needs external forces and dissipation that break the Hamiltonian conservative evolution and lead to compressible phase-space flows, which produce extremely non-uniform measures[1].

The second perspective is the discrete viewpoint of information theory, where the evolution towards equilibrium and entropy saturation are described as the loss of all the information except integrals of motion. On the contrary, to keep a system away from equilibrium, we need to act, producing information and decreasing entropy. Here we make a step in synthesis of the two approaches, asking: what is the informational manifestation of nonuniform turbulent measures? Such measures are expected to have a low entropy whose limit is set by an interplay between interaction on the one hand and discreteness, coarse-graining or finite resolution on the other hand. We shall look at turbulence from the viewpoint of the mutual information (MI), which measures effective correlations between different degrees of freedom. To keep a system away from equilibrium, environment extracts entropy thus producing information --- where is this information encoded?

Here we consider turbulent systems, which can be treated perturbatively as long as their statistics is close to Gaussian, such as weak wave turbulence (similar approach can be applied to a passive scalar[2] and other systems). We show that the {MI between wave modes} is encoded in cumulants (not described by the traditional description in terms of occupation numbers[3,4].

The information production builds higher and higher correlations, which concentrate sharper and sharper on the resonant surfaces, driving the distribution towards a singular measure. When nonlinearity is small, we show that the entropy decay is due to the triple moment concentrating on the three-wave resonance surface. It is unclear yet how to describe the long-time asymptotic of the entropy decay. When turbulence is driven by a random force, which provides for a phase-space diffusion and smears singularities, the entropy must saturate at a finite value, but the difference with Gaussian random-phase approximation can be large when the Reynolds number is large.

Our consideration of the mutual information growth allows solving the old puzzle: why the direction of the formation of the turbulent spectrum is determined by the energetic capacity? We show that this is so because the direction is determined by the directionality of the information transfer between modes and the scaling of the mutual information between modes scales as energetic capacity.

Let us briefly compare our findings with similar effects near thermal equilibrium. In the simplest case of the linear response of a dilute gas, non-equilibrium anomalies in cumulants appear as long-distance divergences due to Dorfman-Cohen memory effects in multiple collisions[5]. Non-equilibrium build-up of spatio-temporal correlation is a counterpart to our spectral singularities.

Another analogy is with many-body localization, where phase correlations prevent thermalization and keep the system in a low-entropy state.

There is a vast literature devoted to cumulant anomalies away from equilibrium, complementarity of information theory and singular measures must lead to a unified approach to these anomalies.

We conclude reiterating our main results: the probability distribution of weak wave turbulence is very far from Gaussian, the mutual information is substantial for resonant modes.

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