Vlasov-Poisson in cosmology

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What is the nature of dark matter and dark energy, which together make up about 95% of the Universe's energy content? One way to address such questions is to investigate the evolution of cosmic structures on the largest observable length scales (Fig. 1).

Dark matter constitutes the bulk part of the overall matter distribution on cosmic scales. Apart from gravitational interactions, dark matter appears to be extremely weakly interacting, thereby justifying the validity of the collisionless limit on the considered scales. The gravitational evolution of such a collisionless medium is governed by the cosmic Vlasov-Poisson equations, which describe how the dark-matter distribution evolves in the six-dimensional phase-space. Noteworthy, the gravitational collapse of dark matter leads to regions with extreme densities.

The cold dark matter distribution comes with an initially vanishing (thermal) velocity dispersion and occupies, at all times, only a three-dimensional sheet in 6D phase-space. The evolution of this dark-matter sheet can be conveniently parametrized by the Lagrangian map $\boldsymbol{q} \longmapsto \boldsymbol{x}(\boldsymbol{q},\tau)$, which encapsulates the matter trajectories from initial position $\boldsymbol{q}$ to current position $\boldsymbol{x}(\boldsymbol{q},\tau)$ with associated velocity $\boldsymbol{v} = \dot{\boldsymbol{x}}(\boldsymbol{q},\tau)$. Here, τ is a dimensionless time-variable related to the overall expansion of the Universe, and the overdot denotes the corresponding convective time derivative. Using this, the Vlasov-Poisson equations for dark matter take the form

$$\ddot{\boldsymbol{x}} + \frac{2}{3\tau}\dot{\boldsymbol{x}} = -\frac{2}{3\tau}\nabla_x \varphi\,, \qquad \nabla_x^2 \varphi = \frac{\varrho - 1}{\tau}, \tag{1a}$$

where the density ϱ is expressed using the Dirac-delta δ_{D},

$$\varrho = \varrho\big(\boldsymbol{x}(\boldsymbol{q},\tau)\big) = \int \delta_{\mathrm{D}}\,[\boldsymbol{x}(\boldsymbol{q},\tau) - \boldsymbol{x}(\boldsymbol{q}',\tau)]\,\mathrm{d}^3 q'. \tag{1b}$$

It is standard to solve these equations numerically with discretized N-particle approximations, using cosmological N-body simulations (see e.g. [2] for a review). Recently, also phase-space tessellation methods are used to achieve numerically the continuum limit [1, 3].

Analytically or semi-analytically, progress on solving the highly non-linear Eqs. (1) to arbitrary high precision was slow until fairly recently.

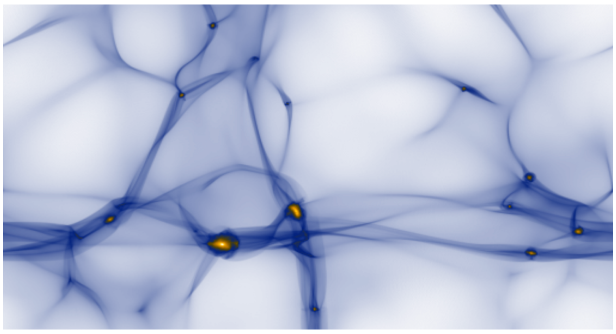

Fig. 1. Simulation result of the dark-matter distribution on cosmic scales in our Universe (here: $l_{\mathrm{box}} \sim 10^{22}$m) [1].

Specifically, the first non-trivial analytical solutions of Eqs. (1) have been reported and tested in Refs. [4-6]. Although these solutions capture very accurately the emergence of the first density singularity, they become invalid immediately afterwards.

We have studied in detail the dark-matter phase-space at times shortly after the appearance of the first singular density, by employing ultra-high resolution N-body simulations and novel theoretical methods [7]. Fig. 2a shows the respective phase-space in one space-dimension, featuring single- and multi-beam regions that are spatially separated by infinite densities (Fig. 2b). At the same time, Fig. 2c shows the corresponding acceleration of particles, displaying four non-differentiable sharp features which is due to particles experiencing infinite densities. Fig. 2d shows the sudden, non-analytic movement of a particle as a consequence of an asymmetry in the initial conditions – which kicks in only after the first density singularity.

Theoretical and numerical tools agree to high precision, thereby providing us deep insight into the skeleton of dynamical concentrations. Applications of our methods to plasma problems, such as the bump-on-tail instability (e.g. [8]) or warm plasmas will be discussed.

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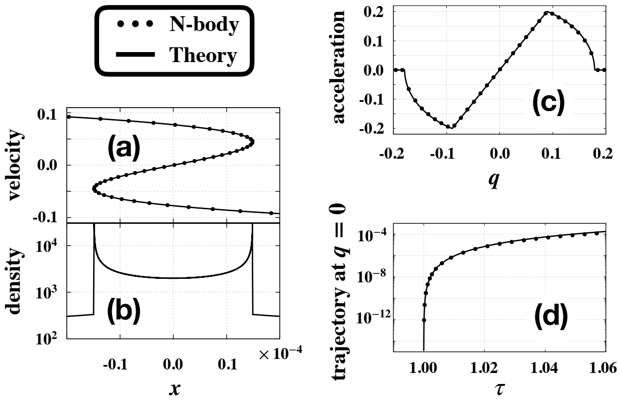

Fig. 2. Numerical (dotted lines) and fully analytical (solid lines) predictions of the phase-space after the first density singularity (at τ=1) with 1D initial data [7].

References

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