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Asymptotic solutions of Fokker-Planck equations for a periodically driven plasma

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Collective dynamics of a periodically driven plasma is of immense importance in various problems, one of them being Paul traps in which charged particles are confined using time-periodic spatially non-uniform electric fields [1]. The distribution function for a collisional plasma in this device was earlier obtained by solving the Vlasov equation, and found to be time-periodic only for certain specific choice of initial distribution functions [2]. In particular, it was shown using Hamiltonian Averaging Theory that there are infinite number of periodic solutions and only one of them matches with the prediction of conventional ponderomotive theory [3].

In the presence of collisions, collective dynamics of a statistical system are governed by the Fokker-Planck equation, which usually leads to a unique asymptotic solution irrespective of the initial plasma distribution. An expression for this asymptotic solution is, however, hard to derive in general due to analytical difficulties. An approximate asymptotic solution for the plasma distribution function in the presence of time-periodic spatially nonlinear forcing was derived earlier using perturbation methods [4], and found to be time-periodic. However, it is not clearly known whether these equations can have multiple asymptotic solutions, or if there is only one single solution in the asymptotic limit. Also, the transient dynamics of various initial distributions is also not known for such periodically driven systems, and can have important experimental consequences.

For the case of small damping and diffusion coefficients, we apply averaging theory to the special case when the 1D time-periodic electric field is spatially linear, and show that the averaged dynamics can be represented by a remarkably simple 2D phase portrait (in parameter space), which is independent of the applied field amplitude [5]. In particular, in the 2D phase portrait shown in Fig. 1, we have two regions of initial conditions. From one region, all solutions are unbounded. From the other region, all solutions go to a stable fixed point, which represents a unique time-periodic solution of the plasma distribution function, and the boundary between these two is a parabola [5].

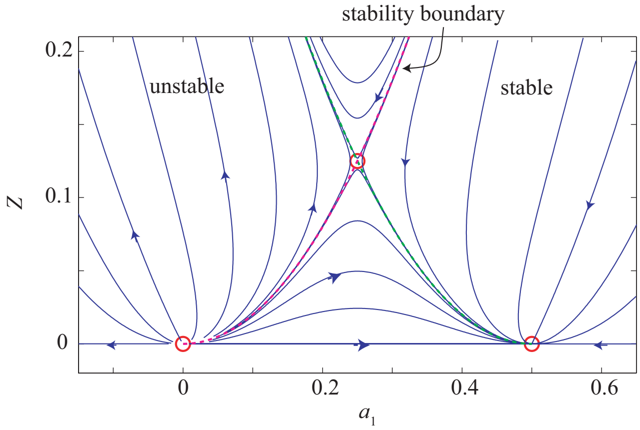

Figure 1 : Unified phase portrait in the reduced parameter space. The fixed points (red circles) are as follows: (i) (0,0), unstable, (ii) (0.5,0) stable, and (iii) (0.25,0.125), saddle. The dashed curve in magenta, denotes the stability boundary.

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