



Turbulence in a Stochastic Magnetic Field

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The problem of understanding the physics of instabilities and turbulence in a stochastic magnetic field is now enshrined among the classics of fusion theory, with a paper trail extending from the 1970s (1). Recent observations that RMPs tend to raise the L-H power threshold (2) have sparked renewed interest in this old topic. It is interesting to note that the overwhelming preponderance of previous studies have focused on anomalous dissipation (i.e. anomalous hyper-resistivity or electron viscosity) but ignored other aspects of the problem.

Here, we present a novel analysis of instability dynamics in a stochastic magnetic field. The prototypical case of a resistive interchange in the collisional regime is the focus of interest. The basic equations are $\text{div } \underline{J} = 0$ and the electrostatic Ohm's Law. The stochastic field requires that all $\underline{B} \cdot \text{grad}$ operators be interpreted as $\underline{B} \cdot \text{grad} = B_0 \cdot \text{grad} + \tilde{B}_\perp \cdot \text{grad}$. The system is reduced to a single stochastic differential equation, with \tilde{B}_\perp as the stochastic variable.

The key point is that this problem is intrinsically a **multi-scale** one, with a finer scale, stochastic \underline{B} field coupled to a larger-scale model envelope. The method of averaging is used to derive an envelope equation, which includes the interaction of fine-scale potential variations with the fine-scale magnetic field. The key physics here is that maintaining quasi-neutrality ($\text{div } \underline{J} = 0$) on all scales forces the appearance of perpendicular currents \underline{J}_\perp , and thus convective cell flows. This is a consequence of maintaining charge balance. Parallel current convergence $\text{div}_\parallel \underline{J}_\parallel \neq 0$, due to small-scale \underline{b} , necessitate the consideration of convective cells. These cells may be thought of as fast interchange modes. The microscale cells drive turbulent viscosity and diffusion, which we calculate. These may be a cause of RMP-induced pump-out. These results are in **marked** contrast to test particle theories, such as that of Rechester and Rosenbluth (3). In fact, our theory is more in the spirit of the analysis by Kadomtsev and Pogutse (4), which derives unexpected results by enforcing $\text{div } \underline{Q} = 0$ on all scales. More generally, this analysis shows that an

ensemble of externally induced magnetic perturbations will necessarily induce an ensemble of cells.

Ultimately, we show how turbulent transport and stochastic bending modify the large-scale cell. Correlations of small-scale potential with stochastic \tilde{B} are of critical importance. Several predictions — testable by computer simulation — are presented. We also discuss how this analysis differs from and extends the extensive ancient history of this subject.

This research was supported by U.S. DOE, Shanghai Jiaotong University, and CNNC, via SWIP.

References

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