Based on a long-distance Coulomb interaction of charged particles in the potential \( U(r) = \frac{\kappa}{r} \), the plasma kinetic equations always meet the divergence because Rutherford differential cross section
\[
\sigma(g, \theta) = \frac{\kappa^2}{4\mu^2 g^4 \sin^4(\theta/2)} = \frac{\kappa^2}{(m\Delta v)^4}
\]
has a singularity at \( m\Delta v = 0 \) here \( \mu \) is reduced mass and \( g \) is the relative velocity of charged particles.

Usually, a cutoff variable should be introduced in order to remove the singularity. The traditional way is to make a cutoff either on impact parameter \( b \) [1] or scattering angle \( \theta \) [2]. A third cutoff variable \( \Delta v \) was introduced for removing the singularity [3] [4].

This presentation will compare the differences of the three kinds of cutoff variables, including impact parameter \( b \), scattering angle \( \theta \) and velocity change \( \Delta v \).

It is shown that the singularity at \( \Delta v = 0 \) cannot be removed by a cutoff on small scattering angle \( \theta (\theta \leq \theta_{min}) \) unless the relative velocity \( g \) is constant. However, in plasma physics, \( g \) can vary from zero to infinite due to varied field particle velocity \( v_p \) even if the test particle velocity \( v \) is a constant. Obviously, the singularity still exists at \( g = 0 \) after the cutoff on \( \theta_{min} \) made. The cutoff on scattering angle \( \theta \leq \theta_{min} \) cannot remove the weak collision effects with both smaller \( g \) and larger \( \theta \).

In fact, scattering angle \( \theta \) has already been proved mathematically to be an incorrect cutoff variable [5]. Similarly, the singularity at \( \Delta v = 0 \) cannot be removed by a cutoff on large impact parameter \( b \) (\( b \geq b_{max} \)) unless \( g \) is constant. Obviously, the singularity still exists at \( g = 0 \) after the cutoff on \( b_{max} \) made. The cutoff on impact parameter \( b \geq b_{max} \) cannot remove the weak collision with smaller \( g \) and smaller \( b \).

Recently, we claim the impact parameter \( b \) is an incorrect cutoff variable. The traditional practice of making the cutoff on small impact parameter \( b \leq b_{min} \) is a total mistake. Small impact parameter is not the reason of divergence as Landau once pointed out [2] 'if the exact formulae are used, then there would, of course, be no divergence at small b'. Landau’s predication is proved by our exact mathematical calculation [6].

The velocity change \( \Delta v \) is so far the only correct cutoff variable that is mathematically proved [4].

Consider a test particle \( \alpha \) in a collection of \( \beta \) particles with a Maxwellian distribution, the nth order Fokker-Planck coefficients are defined as the integral
\[
\left( \Delta v_{k}^{n-2(j+k)} \Delta v_{l}^{2j} \Delta v_{\perp}^{2l} \right)
\]
\[
\nu(\alpha v_{th})^{n}
\]
\[
= \sum_{i=0}^{\infty} \frac{\delta_{n}(y_{min}, u)}{(j-k-i)!}
\]
Where the set of functions \( q_{n}(y_{min}, u) \) is defined as
\[
q_{n}(y_{min}, u) = \frac{2}{\sqrt{\pi}} \int_{y_{min}}^{\infty} e^{-(y+\nu^{-2})} y^{n} dy
\]
\[
\nu = n_{\alpha} v_{th} m_{\alpha} / (2 \mu k_{B} T_{\beta})^{2}, \ y_{min} = \Delta v_{min} / (\nu v_{th}), \ a = 2\pi / m_{\alpha}
\]
The energy transfer moments are defined as
\[
\langle \epsilon \rangle = \int \epsilon f_{0}(v_{\alpha}, T_{\alpha}) g \sin 0 \theta d\theta d\phi dp dv_{\beta}
\]
The arbitrary high order of energy transfer rate can be derived by the cutoff \( \Delta v \geq \Delta v_{min} \) as
\[
\langle \epsilon \rangle = \int \epsilon f_{0}(v_{\alpha}, T_{\alpha}) g \sin 0 \theta d\theta d\phi dp dv_{\beta}
\]
\[
\omega = n_{\alpha} n_{\beta} v_{\alpha} p_{\perp} / (m_{\alpha} k / 2 \mu k_{B} T_{\beta})^{2}, \ \bar{\epsilon} = 4\mu^{2} k_{B} / (T_{\beta} - T_{\alpha}) m_{\alpha} p_{\beta}
\]

References