

# Turbulent energy flux: applications to drag reduction in MHD turbulence and in dynamo

Mahendra Kumar Verma

<sup>1</sup> Department of physics, Indian Institute of Technology Kanpur, Kanpur  
 mkv@iitk.ac.in (speaker):

In hydrodynamic turbulence, the kinetic energy injected at large scales cascades to the inertial range and then to small scales. According to Kolmogorov’s theory of turbulence, the above energy transfers lead to a constant kinetic energy flux in the inertial range [1]. In contrast, in magnetohydrodynamic (MHD) turbulence, a fraction of kinetic energy is transferred to the magnetic energy. This energy transfer to the magnetic field is responsible for the dynamo mechanism [2,3].

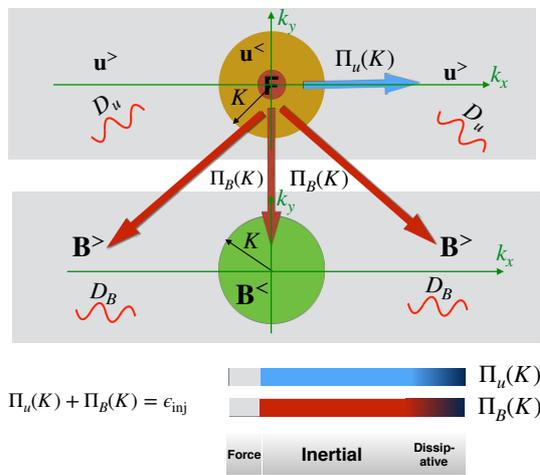


Figure 1: The yellow (green) sphere represent the velocity  $\mathbf{u}$  (magnetic  $\mathbf{B}$ ) Fourier modes inside a wavenumber sphere of radius  $K$ .  $\Pi_u(K)$  is the kinetic energy flux for the sphere, and  $\Pi_B(K)$  is the net energy transfer from  $\mathbf{u}$  modes inside the sphere to all the  $\mathbf{B}$  modes. The external force injects kinetic energy into the small red sphere with the rate of  $\epsilon_{inj}$ . The energy fluxes are dissipated with the dissipation rates  $D_u$  and  $D_B$ . In the inertial range,  $\Pi_u(K) + \Pi_B(K) \approx \epsilon_{inj}$ . Figure taken from Verma et al. [4].

In a magnetofluid, let us assume that we force the large scale modes of the velocity field and inject energy  $\epsilon_{inj}$  per unit time. A fraction of this energy goes to small scales of the velocity field as a kinetic energy  $\Pi_u(K)$ , where  $K$  is the radius of the wavenumber sphere (see

Figure 1). However, a fraction of the energy goes to the magnetic field (both small and large scales) as  $\Pi_B(K)$ . In the inertial range,  $\Pi_u(K) + \Pi_B(K) \approx \epsilon_{inj}$ . This is because of lack of dissipation [4].

Due to the above velocity-to-magnetic field transfer, for the same kinetic energy injection rate as in hydrodynamic turbulence, the kinetic energy flux in MHD turbulence is lower than its hydrodynamic counterpart. This leads to a relative weakening of the nonlinear term  $\mathbf{u} \cdot \nabla \mathbf{u}$  (where  $\mathbf{u}$  is the velocity field), and hence weaker turbulent drag. However, the rms value of the velocity field is increased, with  $\mathbf{u} \cdot \nabla \mathbf{u}$  taking a lower value due to a decoherence among the phases of the velocity Fourier modes. We verify the above using shell model simulations of hydrodynamic and MHD turbulence [4]. We also show similar turbulent drag reduction in quasi-static MHD turbulence [5].

Similar drag reduction is also at work in turbulent flows with dilute polymers. Polymers are like springs, hence they take a fraction of kinetic energy flux, as in in MHD turbulence [4].

The aforementioned drag reduction is important for engineering flows, as well as for the astrophysical applications, such as planetary and stellar dynamos.

### References:

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