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Theory and algorithms for metaplectic modeling of caustics in ray-tracing codes

N. A. Lopez[1]

[1] Department of Astrophysical Sciences, Princeton University

e-mail (speaker): nalopez@princeton.edu

Radiofrequency (RF) waves are widely used in nuclear fusion experiments, providing auxiliary heating and current drive in magnetic confinement fusion and directly or indirectly compressing the fuel pellet in inertial confinement fusion. Due to this ubiquity, the accurate and efficient simulation of wave propagation in plasma is crucial.

A common technique for modeling wave propagation is the ray-tracing method, in which the wave amplitude is evolved along certain spatial trajectories (rays) to determine the resultant intensity distribution. However, since ray tracing relies on the geometrical-optics (GO) approximation, it will fail at caustics, such as cutoffs and focal points. At these points, GO erroneously predicts the wave intensity to be infinite. It is commonly believed that accurate modeling of waves in such regions is impossible without computationally expensive full-wave simulations.

In this talk, I shall discuss a recently developed less expensive alternative for simulating waves near caustics, called metaplectic geometrical optics (MGO) [1, 2]. Rather than evolving the wavefield in the usual coordinate or spectral representations, MGO uses a mixed representation that is chosen as the optimal representation to avoid caustic singularities. We develop this optimal representation based on geometric observations of the wave equation near caustics. After obtaining an expression for the wavefield in the mixed representation, the result is mapped back into the original spatial coordinates using the metaplectic transform, which is a generalization of the Fourier transform.

Besides outlining the basic theory of MGO, this talk shall also present new algorithms that have recently been developed to form the backbone of an MGO-based ray-tracing code that remains accurate at caustics. These algorithms include a fast metaplectic transform algorithm that is exactly unitary [3, 4], and a steepest-descent quadrature rule based on the Freud polynomials for computing the highly oscillatory catastrophe integrals that often arise in MGO [5].

Lastly, I shall present results of applying the MGO formalism to accurately compute a series of classic caustic examples, including the fold (Airy) caustic that occurs near a cutoff, the cusp (Pearcey) caustic that occurs near a focal point, and the fold caustic network that governs the quantum harmonic oscillator. These results open a path towards speeding up radiofrequency-wave simulations for fusion applications and modeling laser-plasma interactions.

References

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