Non-relativistic Stöermer problem in a dipole plasma (with drag)

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The motion of a charged particle in a dipole magnetic field has been a problem of widespread interest, with its origins rooted in the twentieth century, since the discovery of cosmic rays in the Earth's magnetosphere. Extensive studies in the problem were carried out by Carl Stöermer [1], and the eponymous problem was found to form a non-integrable system, in particular, it gives a chaotic system. This problem has also been studied in the restricted case, where particle trajectories are restricted to a fixed radial distance, and the integrable dynamics depict periodic and non-periodic orbits [2].

A natural extension to this problem is to improve upon two key assumptions undertaken in the classical Stöermer problem. We first attempt to account for the effect of the charged particles in the magnetospheric plasma on the electron trajectories as an extension of the single particle formulation in the original problem. Second, we account for the collisional drag force by the neutral particles in the dipole plasma on the electron trajectories. These improvements should provide a more realistic understanding of particle trajectories in a magnetospheric plasma.

Presently, our system of interest is a microwave (2.45 GHz) generated compact dipole Argon plasma developed in the laboratory which uses a single permanent magnet (NdFeB) to generate the magnetic field. Extensive temperature and density measurements have been carried out in the dipole plasma, as well as a characterization of the optical emissivity [3-4], primarily at a neutral pressure of 0.4-2.0 mTorr, and a wave power of 200-300 W. To describe the particle trajectories (in the no-drag approximation) in this plasma, we may solve the following equation of motion given by the Lorentz force equation:

$$m_e \frac{\partial \boldsymbol{u_e}}{\partial t} = -e(\boldsymbol{E} + \boldsymbol{u_e} \times \boldsymbol{B})$$

where $\boldsymbol{u_e}(\boldsymbol{r}, t)$ is the average electron velocity in the plasma, which lies in the non-relativistic regime. Note that, although the electric and magnetic field are also composed of contributions from that of the steady-state continuous mode microwaves used to generate the plasma, their magnitudes are several orders smaller than the plasma electric field or dipole magnetic field, and hence may be safely ignored. The electric field is obtained by the gradient of the space potential, whose profiles are reconstructed for all space from experimental data obtained using an emissive probe at certain radial and angular locations in the plasma [3]. We choose parametrized smooth functions to fit the experimental data, thus accurately representing the transition from a pinched parabolic profile near the magnet to a more circularly symmetric profile further away in the (r, θ) plots [3], which is then converted to a surface of revolution to represent the φ symmetry in the configuration. The Lorentz force equation is solved numerically both by direct integration using a Runge-Kutta procedures, but also formulated analytically using a Lagrangian-Hamiltonian formulation, and the trajectories are studied. Assuming a dipole magnetic field oriented along the positive z-axis and of dipole moment m, we obtain the following Lagrangian (in spherical coordinates) for the system:

$$L = \frac{1}{2} m_e\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\varphi}^2 sin^2\theta\right) + e\Phi + \frac{k\dot{\varphi} sin^2\theta}{r}$$

where Φ is the space potential of the plasma and $k = \frac{e\mu_0 m}{4\pi}$ is a lumped constant. The conjugate momenta and Hamiltonian is constructed subsequently, and the constants of motion are identified. The trajectories are then obtained in phase space numerically and studied in the equatorial and polar limits, as well as characterized into periodic and chaotic orbits.

To account for the collisional drag in the plasma, namely for electron-neutral collisions, we write the equation of motion using the Langevin equation as follows:

$$m_e \frac{\partial \boldsymbol{u_e}}{\partial t} = -e(\boldsymbol{E} + \boldsymbol{u_e} \times \boldsymbol{B}) - m_e \nu_c \boldsymbol{u_e}$$

where $\nu_c(\boldsymbol{r})$ is the effective collision frequency. In our system of choice, this may be calculated in terms of the electron temperature and the neutral pressure in the plasma [3]. The profiles of the electron temperature in all space are again reconstructed from experimental data [3] in a similar fashion as that used for the space potential. Owing to the inclusion of the dissipative force term in the equation of motion, a Lagrangian-Hamiltonian formulation of the problem becomes considerably difficult. We thus, attempt to solve this equation numerically using a Runge-Kutta procedure, and the trajectories of the particles are studied. The effect of drag is characterized and the chaotic particle trajectories are intended to be investigated using standard non-linear dynamical techniques [5].

References:

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