5th Asia-Pacific Conference on Plasma Physics, 26 Sept-1Oct, 2021, Remote e-conference

Existence of ideal magnetofluidostatic equilibria with nonconstant pressure in asymmetric domains

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We study the possibility of constructing steady magnetic fields satisfying the force balance equation of ideal magnetohydrodynamics with tangential boundary conditions in asymmetric confinement vessels, i.e., bounded regions that are not invariant under continuous Euclidean isometries (translations, rotations, or their combination). This problem usually arises in the design of next-generation fusion reactors (stellarators), where the field line twist required to trap charged particles is achieved by appropriately shaped external coils. In this setting, the existence of a so-called quasisymmetry (the invariance of field strength along a direction in space), is expected to enhance confinement along the magnetic field. Indeed, quasisymmetry leads to conservation of the canonical momentum associated with the symmetry (in the sense of Noether's theorem) of the guiding-center Hamiltonian, which is a function of magnetic field strength. The ideal force balance equation in anisotropic magnetohydrodynamics has the form

$$(\nabla\times\boldsymbol{B})\times\boldsymbol{B}=\nabla\cdot\Pi,\quad \nabla\cdot\boldsymbol{B}=0\quad \text{in }\Omega,\quad (1a)$$

$$\boldsymbol{B}\cdot\boldsymbol{n}=0\quad \text{on }\partial\Omega.\quad (1b)$$

Here, $\boldsymbol{B}$ is the magnetic field, $\Omega\subset\mathbb{R}^3$ a smoothly bounded domain, $\boldsymbol{n}$ the unit outward normal to the bounding surface $\partial\Omega$, and Π the symmetric pressure tensor with Cartesian components

$$\Pi^{ij}=P_\perp\delta^{ij}+\frac{P_\parallel-P_\perp}{B^2}B^iB^j,\quad i,j=1,2,3,\quad (2)$$

where $P_\perp$ and $P_\parallel$ are the perpendicular and parallel pressures respectively. Notice that isotropic equilibria correspond to the case $P_\perp=P_\parallel=P$, with P the scalar pressure. The magnetic field $\boldsymbol{B}$ is quasisymmetric along a vector field $\boldsymbol{u}$ provided that

$$\boldsymbol{B}\times\boldsymbol{u}=\nabla\psi,\quad \boldsymbol{u}\cdot\nabla B=0,\quad \nabla\cdot\boldsymbol{u}=0,\quad (3)$$

where ψ is any function. Observe that symmetric magnetic fields are a special case of quasisymmetry where $\boldsymbol{u}=\boldsymbol{a}+\boldsymbol{b}\times\boldsymbol{x}$, with $\boldsymbol{a},\boldsymbol{b}\in\mathbb{R}^3$ constant vectors, is the generator of Euclidean isometries.

Aim of the present work is to elucidate the existence of regular solutions of system (1) with prescribed symmetry properties such as (3) in asymmetric bounded domains. In this regard, the following results are shown.

(i) In the isotropic case $P_\perp=P_\parallel=P$, it is possible to construct solutions of (1) in an asymmetric bounded domain with a nonconstant pressure if one relaxes the standard assumption that the vessel boundary corresponds to a pressure isosurface [1]. In particular, we exhibit a smooth solution that possesses a Euclidean symmetry and yet solves the boundary value problem in an asymmetric ellipsoidal domain while sustaining a nonvanishing pressure gradient. Nonetheless, the problem of existence of asymmetric solutions in bounded domains remains open (see also [2,3] on the existence of asymmetric solutions in the case without boundary conditions).

(ii) In the anisotropic case, first we derive a system of two coupled nonlinear first-order partial differential equations expressing quasisymmetric solutions of eqs. (1) and (3). Then, we devise a method to construct local quasisymmetric solutions of eqs. (1) and (3) explicitly in asymmetric toroidal domains. These solutions are local in the sense they hold in a neighborhood $U\subseteq\Omega$ and satisfy boundary conditions on a portion of the boundary $\partial U\cap\partial\Omega\neq\emptyset$. Figure 1 shows a quasisymmetric solution constructed as described above (the locality of the solution is evident from the discontinuity at $\phi=0$, with ϕ the toroidal angle). Finally, we find that the existence of global solutions depends on the possibility of extending local solutions consistently around the torus.

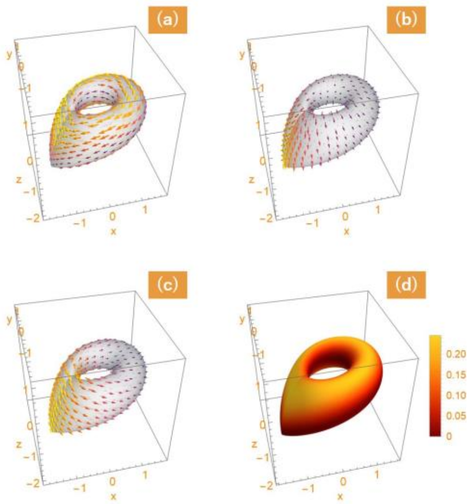

Figure 1: Plot of a locally quasisymmetric field (a), its current (b), quasisymmetry (c), and field strength (d).

References

[1] N. Sato, "*Symmetric ideal magnetofluidostatic equilibria with nonvanishing pressure gradients in asymmetric confinement vessels*", Phys. Plasmas **27**, 122507 (2020).

[2] N. Sato, "*Existence of ideal magnetofluid equilibria without continuous Euclidean symmetries*", Plasma Phys. Control. Fus. **61**, 125013 (2019).

[3] N. Sato, "*Local representation and construction of Beltrami fields*", Physica D: Nonlinear Phenomena **391**, pp. 8-16 (2019).

[4] N. Sato, "*Quasisymmetric equilibria in anisotropic magnetohydrodynamics*" (in preparation).