

Bidirectional cascades in turbulent flows

Alexandros Alexakis¹,

¹ Laboratoire de Physique de l'Ecole Normale Supérieure, ENS,
 Université PSL, CNRS, Sorbonne Université, Université de Paris
 e-mail: alexakis@phys.ens.fr

Recent studies have demonstrated that turbulent flows in the presence of confinement, rotation, stratification, magnetic fields or other symmetry breaking mechanisms can display bidirectional cascades: a simultaneous cascade of energy to both large and small scales. Figure 1 displays the energy flux in a thin layer forced at scale ℓ that displays such a behavior. As the layer height h (measured by the parameter $\lambda \approx \ell/h$) is varied the system transitions from a state that all energy cascades to small scales (indicated by the positive flux for dark lines) to a state that all energy cascades to the large scales (indicated by the negative flux for the light green lines). Such bidirectional cascades have been observed in planetary flows¹⁻³ and control the fate of the injected energy that could lead to the formation of large scale coherent structures like jets and hurricanes if the cascade is strongly inverse to fast dissipation and heat production at the smallest viscous scales if the cascade is forward.

The fraction of energy that cascades to large scales to the total injected energy depends on the system control parameters like the height of the layer, rotation or the strength of the magnetic field. Numerical studies have shown that the inverse cascade can disappear at some "critical" value of the control parameter so that all energy cascades to the small scales. These observations generate a number of theoretical questions. Do bidirectional cascades survive the infinite Re and infinite domain limit? Or are they transient phenomena? How criticality emerges in such systems? What is the turbulence behavior near the critical points? Are they universal? Should we expect a first or a second order phase transition? All these are questions that recent turbulent research is trying to address.

In this talk I will review our present day understanding of bidirectional cascades as presented in [4]. The presentation will be based on some of the simplest examples examined in the literature that display bidirectional cascades. I will try to answer how forward and inverse cascades can coexist and if they can survive

the infinite Reynolds and infinite domain limit. I will attempt to classify all possible transitions from forward to inverse cascades as the control parameters are varied and discuss what type of transitions. Finally I will present open questions in the field.

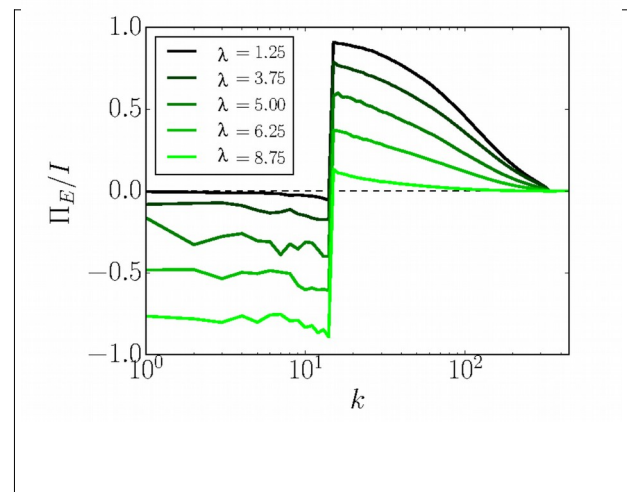


Fig 1 Energy flux in a thin layer for different values of the layer height h measured by $\lambda \approx \ell/h$. As the Layer height is varied the forward cascade (positive flux) transitions to an inverse cascade (negative flux). The figure is taken from [5].

References

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