Zonostrophy, which was discovered by Balk et al. (1991) and Balk (1991), is an invariant for Rossby-wave turbulence governed by the two-dimensional quasi-geostrophic equation on a beta-plane (also known as the Charney-Hasegawa-Mima, or CHM equation). Zonostrophy is useful for explaining the anisotropic energy cascade that favors zonally elongated structures (Balk, 2005).

In this talk, after briefly describing the conservation property of zonostrophy and its effect on the pattern formation, I consider Rossby-wave turbulence in the limit of large-scale flows (long-wave limit), in which \( L_p / L \to 0 \). Here, \( L_p \) is the Rossby radius of deformation and \( L \) is the characteristic length scale of the flow. In this case, the ratio of the linear term that originates from the beta-term to the nonlinear terms is estimated by a dimensionless number, \( \gamma = \beta L_p^2 / U \), where \( \beta \) is the latitudinal gradient of the Coriolis parameter and \( U \) is the characteristic velocity scale.

I first show the asymptotic expression for zonostrophy in the long-wave limit. This expression was found by Saito & Ishioka (2013) and referred to as semi-action (Connaughton et al., 2015). Next, I show the results of numerical simulations of the CHM equation in the long-wave limit conducted to examine the conservation of semi-action and its effect on the pattern formation (Figure 1). As \( \gamma \) increases, the inverse energy cascade becomes more anisotropic. When \( \gamma > 1 \), the anisotropy becomes significant and energy accumulates in a sector where \( |l| > \sqrt{3} |k| \) in the two-dimensional wavenumber space. Here, \( k \) and \( l \) are the longitudinal and latitudinal wavenumbers, respectively. When \( \gamma \) is increased further, the energy concentration on the lines of \( |l| = \sqrt{3} |k| \) is clearly observed. These results are interpreted based on the conservation of semi-action. I also discuss the possible relevance to Rossby-waves in the ocean.

References

Figure 1. Simulation results. (a) Time evolutions of zonostrophy (semi-action) \( Z \) for the cases with \( \gamma = 0 \) (solid red), \( \gamma = 0.25 \) (long dashed green), \( \gamma = 1 \) (short dashed blue), \( \gamma = 5 \) (dashed-dotted cyan), and \( \gamma = 20 \) (dashed-two-dotted purple). Note that \( Z \) is conserved very well when \( \gamma = 5 \) and 20. (b) Angular distribution of energy spectra at the final \( t = 0.4 \) states of the simulations. Note that, for \( \gamma > 1 \), energy accumulates in a sector where \( |l| > \sqrt{3} |k| \), corresponding to the range of azimuthal angle from 60° to 90°. (c) Two-dimensional energy spectrum at \( t = 0.4 \) for \( \gamma = 20 \) averaged over 41 ensemble members. Two black lines in the panel indicate \( |l| = \pm \sqrt{3} |k| \).