Zonostrophy invariant and its relation to pattern formation in CHM turbulence

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Zonostrophy, which was discovered by Balk et al. (1991)[1] and Balk (1991)[2], is an invariant for Rossby-wave turbulence governed by the two-dimensional quasi-geostrophic equation on a beta-plane (also known as the Charney-Hasegawa-Mima, or CHM equation). Zonostrophy is useful for explaining the anisotropic energy cascade that favors zonally elongated structures (Balk, 2005)[3].

In this talk, after briefly describing the conservation property of zonostrophy and its effect on the pattern formation, I consider Rossby-wave turbulence in the limit of large-scale flows (long-wave limit), in which $L_D/L \to 0$. Here, L_D is the Rossby radius of deformation and L is the characteristic length scale of the flow. In this case, the ratio of the linear term that originates from the beta-term to the nonlinear terms is estimated by a dimensionless number, $\gamma = \beta L_D^2/U$, where β is the latitudinal gradient of the Coriolis parameter and U is the characteristic velocity scale.

I first show the asymptotic expression for zonostrophy in the long-wave limit. This expression was found by Saito & Ishioka (2013)[4], and referred to as semi-action (Connaughton et al., 2015)[5]. Next, I show the results of numerical simulations of the CHM equation in the long-wave limit conducted to examine the conservation of semi-action and its effect on the pattern formation (Figure 1). As γ increases, the inverse energy cascade becomes more anisotropic. When $\gamma > 1$, the anisotropy becomes significant and energy accumulates in a sector where $|l| > \sqrt{3}\,|k|$ in the two-dimensional wavenumber space. Here, k and l are the longitudinal and latitudinal wavenumbers, respectively. When γ is increased further, the energy concentration on the lines of $|l| = \sqrt{3}\,|k|$ is clearly observed. These results are interpreted based on the conservation of semi-action. I also discuss the possible relevance to Rossby-waves in the ocean.

References

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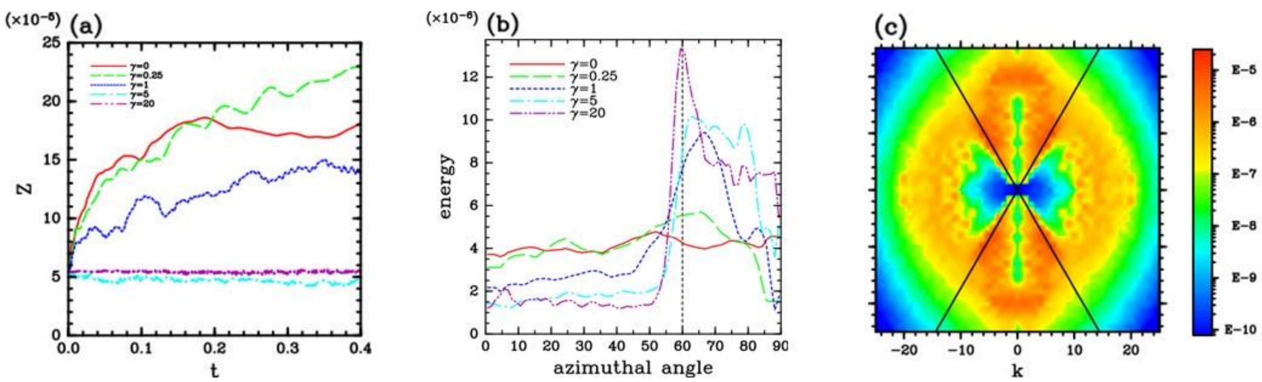

Figure 1. Simulation results. (a) Time evolutions of zonostrophy (semi-action) Z for the cases with $\gamma = 0$ (solid red), $\gamma = 0.25$ (long dashed green), $\gamma = 1$ (short dashed blue), $\gamma = 5$ (dashed-dotted cyan), and $\gamma = 20$ (dashed-two-dotted purple). Note that Z is conserved very well when $\gamma = 5$ and 20. (b) Angular distribution of energy spectra at the final (t = 0.4) states of the simulations. Note that, for $\gamma > 1$, energy accumulates in a sector where $|l| > \sqrt{3}\,|k|$, corresponding to the range of azimuthal angle from 60° to 90°. (c) Two-dimensional energy spectrum at t = 0.4 for γ=20 averaged over 41 ensemble members. Two black lines in the panel indicate $|l| = \pm\sqrt{3}\,|k|$.