Energetic-particle-driven instabilities exhibit a broad range of nonlinear behavior in plasmas, including steady state solutions, frequency chirping, bursting, chaos, etc. It is now recognized that the competition between convective (dynamical friction, also known as drag) and diffusive (scattering) collisions plays a key role in determining the dynamical behavior of resonant wave-particle systems. However, the effects of drag on the ubiquitous quasi-steady solutions have not been thoroughly investigated.

Hence, the electrostatic bump on tail problem is studied analytically in order to determine the effect of drag ($\alpha_{\text{eff}}$) on steady state solutions near marginal stability ($1 - \gamma_d/\gamma_L \ll 1$) when the effective scattering rate across a narrow resonance is large ($\nu_{\text{eff}} \gg \gamma$). In this common tokamak regime, it is rigorously shown that the paradigmatic Berk-Breizman cubic equation for the nonlinear mode evolution reduces to a much simpler differential equation, dubbed the time-local cubic equation, which admits an exact analytic solution [1]. It is found that in addition to increasing the saturation amplitude, drag introduces a shift in the apparent oscillation frequency by modulating the saturated wave envelope, as illustrated in Figure 1. Excellent agreement is found between these analytic results and 1D Vlasov simulations.

Remarkably, a quasilinear diffusion equation for $\delta f$ naturally emerges from the nonlinear system under these conditions, even for a single, isolated resonance. It is found that drag fundamentally changes the structure of the wave-particle resonance, breaking its symmetry and leading to the shifting and splitting of resonance lines [2] seen in Figure 2. In contrast, scattering on its own broadens the resonance in a symmetric fashion. While the influence of drag is modest when the ratio of effective drag to effective scattering is negligible, it can become substantial when $\alpha_{\text{eff}}/\nu_{\text{eff}} > 0.5$, suggesting that drag should be accounted for in quantitative models of fast-ion-driven instabilities in fusion plasmas. This work was supported by US DOE contracts DE-SC0020337 and DE-AC02-09CH11466.

References

Figure 1. Top: spectrogram of 1D Vlasov simulation with shown parameters. The linear oscillation frequency is shown as the green dashed line. The blue curve shows the time-dependent nonlinear frequency calculated analytically. Bottom: magnitude of the mode amplitude from the same simulation.

Figure 2. Wave-particle resonance functions modified by drag. Top: scattering-dominated regime where small amounts of drag induce an asymmetric shift. Bottom: drag-dominated regime where large amounts of drag split the resonance into several distinct peaks.