# Simulated annealing as a method for stability analysis

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Simulated annealing (SA) is a method to calculate a steady state of a Hamiltonian system by solving time evolution of artificial dynamics derived from the original dynamics. The artificial dynamics is constructed so that the energy of the system changes monotonically while preserving Casimir invariants. SA was first applied to two-dimensional vortex dynamics, and significantly generalized in [1]. SA leads to an energy extremum, which is an equilibrium, on a Casimir leaf. We have demonstrated that SA succeeds to calculate low-beta reduced MHD equilibria in rectangular and cylindrical domains[2, 3], as well as high-beta reduced MHD toroidal equilibria[4].

Since the we find an energy minimum in the case of MHD by the SA, the achieved equilibrium must be stable. Therefore, the SA can be used as a tool for stability analysis. Namely, if we perform SA starting from an initial condition that is a summation of an equilibrium and a small perturbation, and if the SA recovers the original equilibrium, it is stable.

In such a stability analysis via SA, it is important to perturb the original equilibrium without escaping from the Casimir leaf. In [5], we developed a method to perturb the equilibrium while staying on the Casimir leaf. For the low-beta reduced MHD in a cylindrical geometry, the SA equation is derived as

$$\frac{\partial U}{\partial t} = [U, \tilde{\varphi}] + [\psi, \tilde{J}] - \varepsilon \frac{\partial \tilde{J}}{\partial \zeta}, \qquad (1)$$

$$\frac{\partial \psi}{\partial t} = [\psi, \tilde{\varphi}] - \varepsilon \frac{\partial \tilde{\varphi}}{\partial \zeta}, \qquad (2)$$

where $U$ is the vorticity along the ambient magnetic field, $\psi$ is the magnetic flux function, $\varepsilon$ is the inverse aspect ratio, $\zeta$ is the toroidal angle, and $\tilde{\varphi}$ and $\tilde{J}$ are the advection fields that must be chosen so that the energy decreases monotonically[3]. Here, we note that the Casimirs are preserved even if we choose arbitrary $\tilde{\varphi}$ and $\tilde{J}$. The energy can increase in this case, of course. Then we can generate a series of perturbed states by evolving the above equations for any choice of $\tilde{\varphi}$ and $\tilde{J}$.

In [5], we examined stability of cylindrically-symmetric low-beta reduced MHD equilibria with and without poloidal rotations by SA. We reported that the SA does not recover the original equilibria even if it is stable against ideal MHD modes; we obtained stationary states with helical components remaining with small but a finite amplitudes as shown in Fig. 1.

In this presentation, we report why the imaginary part remained. We also report SA results for helically perturbed initial conditions without a corresponding rational surfaces.

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**References**

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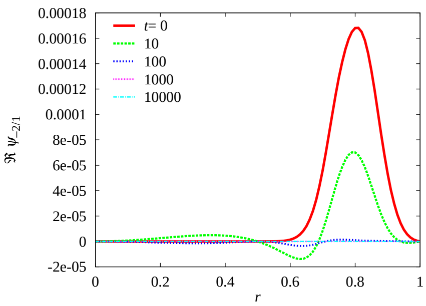
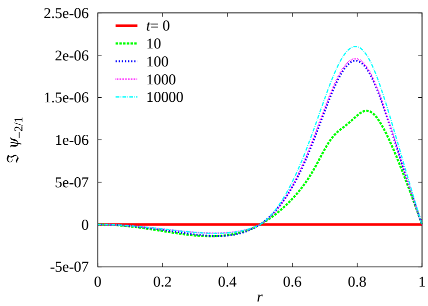


(a) Real part.



(b) Imaginary part.

Figure 1: An example of time evolution of a resonant Fourier component of $\psi$ as a function of the minor radius $r$ by the SA. The imaginary part remains finite.