

Solitons in zonal flows: a study of nonlinear planetary eddies

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The term ‘longevity’ has grown much attention in various aspects of physics. Solitons and solitary waves are major role for (abnormal) long-lived objects. In theory of the integrable system, the stability is guaranteed by existence of conserved quantities, and if there are of infinite number, the lives become enormous. In field theoretical context, topological solitons, i.e., skyrmions, gauged vortices, or monopoles, are stable objects of collective motions whose stability is guaranteed in terms of their topological nature [1]. On the other hand, oscillons [2] or ensemble of Q-balls [3] are the notable examples of the longevity without topological origin. In the planetary atmosphere, there are a large number of phenomena concerning vortices of a nonlinear dynamics. Jupiter’s Red Spot is considered as a solitary wave because of its extraordinary longevity. For the better understanding, several dynamical regimes were proposed so far [4], and the zonal flows were regarded as a kind of guide-rail to inhibit the transport of eddies across the flows. However, we have found the zonal flows play a major role for the longevity, since they are a source for keeping the conserved quantities such as the energy, or the enstrophy.

We use a simple model on the beta-plane in the intermediate geostrophic regime proposed by Williams, Yamagata [5] and also Flierl [6]. This W-Y-F equation has several advantages: (i) The model is based on dynamics of the Korteweg-de Vries (K-dV) or related known integrable models and then, it enables to incorporate many fruits from the studies of the integrable systems. (ii) The simple, 2-dimensional computation brings us the good mathematical intuition for the system. (iii) The model integrates several independent effects concerning the longevity and the confluence of vortices; the synergy has not been examined yet.

W-Y-F equation has connections to a 2+1-dimensional nonlinear wave equation by Zakharov-Kuznetsov (Z-K) [7], which may be seen as a model for the cyclonic shear in the uniform background flow. There are one parameter family of the circular symmetric solutions. In terms of the similarity of the equations, these solutions of the Z-K equation are good initial guess for the anti-cyclonic solutions. By imposing the background shear flow, we are able to get stable solitary waves of anti-cyclonic solutions. For several strength and the direction of flows, we examine the life of the vortices. Also, W-Y-F equation possesses an advection term which realizes the

merger of the vortices; it is a quite distinct feature than the standard integrable systems. This effect certainly contributes the stability of the vortices. We perform the numerical analysis especially the synergy of these effects and explain the origin of the longevity.

For the numerical analysis, we adapt an explicit method with 4th order finite difference stencil for the spatial derivatives and evolve in time with the classical Runge-Kutta 4th order scheme. In the presentation, we also discuss why the zonal flows can support the existence of those vortices with longevity and how they interact with each other. Detail of our analysis will eventually be presented in [8]. This work is supported by JSPS KAKENHI Grant Number JP B20K03278(1).

References

- [1] N.Manton, P.Sutcliffe, “Topological Solitons”, Cambridge Monographs on Mathematical Physics, 2004.
- [2] I. L. Bogolyubsky, V. G. Makhankov, Pisma Zh. Eksp. Teor. Fiz.24, 15 (1976).
- [3] E.J.Copeland, P.M.Saffin,S.-Y.Zhou, Phys.Rev.Lett.113 (2014)231603.
- [4] J.-I. Yano, Y. N. Tsujimura, Dynamics of Atmospheres and Oceans, 11 (1987) 101-129
- [5] G.P.Williams, T.Yamagata, J.Atmos.Sci.,41,453 (1984).
- [6] J.G.Charney, G.R.Flierl, “Oceanic analogue of large-scale atmospheric motion. Evolution of Physical Oceanography” (the Stommel volume), The MIT press, 504-548, 1981.
- [7] V.E. Zakharov, and E.A. Kuznetsov, Sov. Phys. JETP 39(1974), 285-286
- [8] Y.Koike, A.Nakamura, S.Horihata, A.Nishie, K.Obuse,N.Sawado,Y.Suda,K.Toda, in preparation.

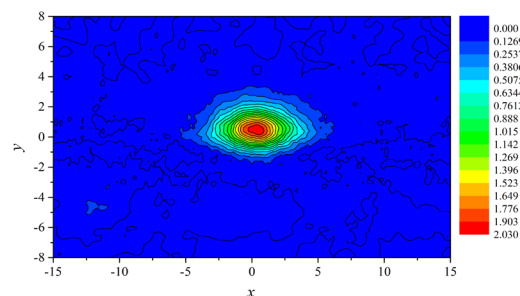


Figure 1. The solution of Red Spot in W-Y-F equation.