

Spherical and β -plane approximation models for two-dimensional turbulence on a rotating sphere

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When investigating fluid dynamics on a rotating sphere, a two-dimensional mathematical model is sometimes considered. Although this model is fairly simple, it shows rich phenomena, such as the appearance of zonal flows similar to those seen on gas giants, and being a foundation of realistic and complex models for planetary atmospheres. However, the study of this model often contains great amount of difficulties in both analytical and numerical aspects, and even the mechanism of zonal flow formation is not fully understood yet. In order to make this two-dimensional model even simpler, therefore, a β -plane approximation, where the flow is confined in a plane with no curvature, and the Coriolis parameter is approximated linearly with respect to the meridional direction (y -coordinate), is often utilised. However, despite being considered in many situations, two-dimensional turbulence on a beta-plane naturally have different properties from those on a rotating sphere, and this could lead us to misunderstand what we really hope to know.

Then in this talk, I consider unforced two-dimensional flows described by two models stated above, and show several examples where clearly different properties are observed. I start with a comparison of long-time behaviour of the flow field, in particular with respect to zonal flow formation. On a sphere, pure zonal flows are formed (Figure 1 (a-1) and (a-2)) [1], whereas on a β -plane the flow stagnates in a quasi-zonal state and not much energy is accumulated in the pure zonal components even after long enough time (Figure 1 (b-1) and (b-2)).

Both systems possess linear wave solutions called Rossby waves, and the dynamics of Rossby waves determine the overall flow dynamics of these systems. Therefore, I next consider nonlinear interactions between Rossby waves with a focus on resonant nonlinear interactions, and show how the nature of the resonant nonlinear interaction involving zonal Rossby waves differs between the two models [1,2]. As another example, the structure of Lyapunov vectors are also compared. Then I suggest cases where we should be careful in utilising a β -plane as a “good” approximation of a rotating sphere, and *vice versa*.

References:

- [1] Kiori Obuse, Shin-ichi Takehiro, and Michio Yamada, “Long-time asymptotic states of forced two-dimensional barotropic incompressible flows on a rotating sphere”, *Physics of Fluids*, **22**(5), (2010), 1-9 056601
- [2] Kiori Obuse and Michio Yamada, “Three-wave resonant interactions and zonal flows in two-dimensional Rossby-Haurwitz wave turbulence on a rotating sphere”, *Phys. Rev. Fluids* **4**, (2019) 024601
- [3] Kiori Obuse and Michio Yamada, “Energy Transfer to Resonant Zonal Rossby Modes in Two-Dimensional Turbulence on a Rotating Sphere”, *J. Phys. Soc. Jpn.*, **89**(6), (2020) 064401

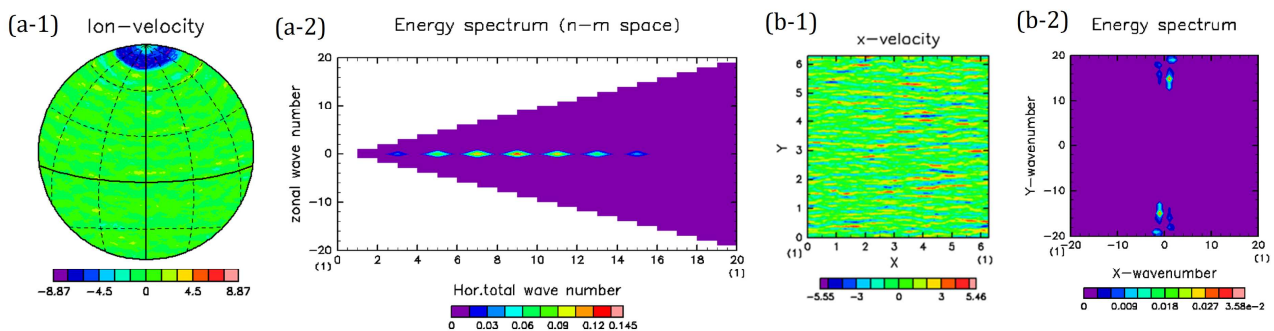


Figure 1:

Comparison of long-time behaviour of flow fields on a sphere and a β -plane, with respect to zonal flow formation.

(a-1): longitudinal velocity on a sphere after long enough time. (a-2): Energy spectrum of the flow field of (a-1). Modes with zonal wavenumber = 0 are the pure zonal modes. Only important parameter region is shown here. (b-1): x-velocity on a β -plane after long enough time. (b-2): Energy spectrum of the flow field of (b-1). Modes with x-wavenumber = 0 are the pure zonal modes. Only important parameter region is shown here.