



Layering or Homogenization — a Matter of Time?

P.H. Diamond and F. Ramirez¹

¹ Department of Physics and CASS, University of California San Diego, La Jolla, USA
e-mail: pdiamond@ucsd.edu

Staircase formation, or layering, is a remarkable example of self-organization and mesoscopic ordering in a turbulent fluid or plasma. However, such staircase structures run contrary to expectations of ‘homogenization’ — the tendency of a closed, bounded eddy or flow to mix scalar concentration, potential vorticity, etc. to a uniform state (1). The expectation of homogenization is underpinned by the Prandtl–Batchelor Theorem (2), which requires a conservative dissipation process for applicability. Usually, the dissipative process involved is related to the divergence of a Fickian flux, with constant diffusivity.

Here, we show that a layered state — characterized by ‘piecewise homogenization’ — is compatible with the requirements for a quasi-stationary state of mixing. Such a layered state occurs naturally when the conservative dissipative flux is bistable, as for passive Cahn–Hilliard–Navier–Stokes flows (3, 4). In that case, the layered state is a target pattern. Such patterns deviate from homogenization only in a finite number of inter-band domain boundaries. We show that the Prandtl–Batchelor Theorem ultimately requires the domain walls to relax, leading to a **globally homogenized state**. The temporal evolution to this state is non-trivial and involves hyper-diffusion. The message of this simple example is that layering is a straightforward consequence of inhomogeneous mixing and (at least) a bistable transport process. It occurs in minimalistic systems, such as **passive scalar advection**. In particular, layering is a simple and generic outcome, which need **not** depend on the frequently involve complex processes of $E \times B$ shearing feedback, avalanches, jamming, etc. And, layering is likely ultimately transient, though persistent on long time scales.

Toward the end of understanding the simplest possible manifestations of layering, we also consider a fixed array of differentially rotating convective cells, in the presence of background diffusion (5). This system naturally manifests layers, with domain boundaries located between cells. Layering results as a consequence of the disparity between the two characteristic time scales of the system — namely those of eddy circulation and diffusion. We probe this simple system and the layered structures it manifests by considering the effects of a cross profile shearing flow (which introduces a third time scale), deposition noise, and statistical variability of cell strength. Results will be discussed.

References

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