

Variable energy fluxes and exact relations in Magnetohydrodynamics turbulence

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Till date, there is no convergence on the MHD turbulence phenomenology. For the energy spectrum, some researchers argue in favour of Kolmogorov-like models ($k^{-5/3}$), while others support models with $k^{-3/2}$ spectrum. Given this controversy, the energy fluxes of MHD turbulence provides valuable insights, and could possibly help us understand the dynamics of MHD turbulence.

MHD turbulence has six fluxes associated with the velocity and magnetic fields. One such flux, $\Pi_{b>}^{u<}(k_0)$, represents the energy flux from the velocity modes inside the sphere of radius k_0 to the magnetic modes outside the sphere [1,2]. Here, the superscript represents the giver field, while the subscript represents the receiver field. The symbols $<$ and $>$ represent the modes inside and outside the sphere respectively. The other fluxes are $\Pi_{u>}^{u<}(k_0)$, $\Pi_{b>}^{b<}(k_0)$, $\Pi_{u>}^{b<}(k_0)$, $\Pi_{b>}^{u<}(k_0)$, and $\Pi_{b>}^{b<}(k_0)$. See Figure 1 for an illustration.

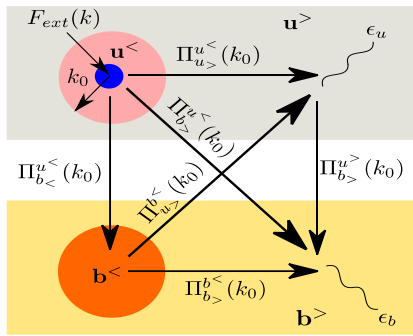


Figure 1: The six fluxes of MHD turbulence. The pink and red spheres are the velocity and magnetic wavenumber spheres of radius k_0 , respectively. $\Pi_{Y>}^{X<}(k_0)$ is the flux from the X modes inside the sphere to the Y modes outside the sphere, while ϵ_u, ϵ_b are the viscous and magnetic dissipation rates respectively.

We performed direct numerical simulation of MHD turbulence and computed the above fluxes [4]. We observe that in the inertial range itself, the energy fluxes of MHD turbulence vary with k due to energy exchanges between the velocity and magnetic fields. This is unlike Kolmogorov's hydrodynamic turbulence where the inertial-range energy flux is constant in k .

Despite the variability of the energy fluxes, these fluxes

satisfy several exact relations due to conservation principles [3,4] (see Figure 2). For example, in the inertial range $\Pi_{u>}^{u<}(k_0) + \Pi_{u>}^{b<}(k_0) + \Pi_{b>}^{u<}(k_0) \approx \epsilon_u$, where ϵ_u is the viscous dissipation rate. In this talk, using numerical simulations, we quantify the variable energy fluxes of MHD turbulence, as well as verify several exact relations.

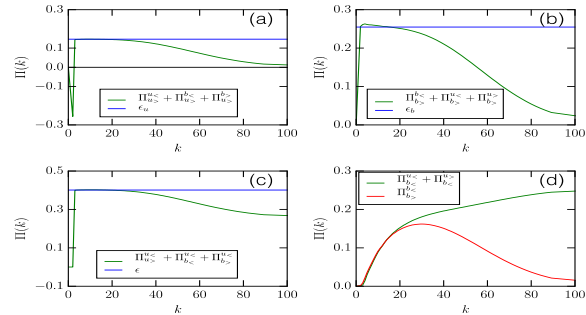


Figure 2: Several combinations of MHD energy fluxes are constant in the inertial range: (a) $\Pi_{u>}^{u<}(k_0) + \Pi_{u>}^{b<}(k_0) + \Pi_{b>}^{u<}(k_0) \approx \epsilon_u$; (b) $\Pi_{b>}^{b<}(k_0) + \Pi_{b>}^{u<}(k_0) + \Pi_{u>}^{b<}(k_0) \approx \epsilon_u$; (c) $\Pi_{u>}^{u<}(k_0) + \Pi_{b>}^{u<}(k_0) + \Pi_{b>}^{u<}(k_0) \approx \epsilon$; (d) $\Pi_{u>}^{u<}(k_0) + \Pi_{b>}^{u<}(k_0) \approx \Pi_{b>}^{b<}(k_0)$.

In addition, we also show that the energy fluxes of Elsässer variables are constant in the inertial range. Consequences of the flux behaviour on spectral theories ($k^{-5/3}$ or $k^{-3/2}$) will be discussed [3,4].

References

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