

## Normal mode spectrum of Multi-Region relaxed Magnetohydrodynamics

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It is well known that the nature of the ideal -MHD equilibria in three dimensional plasma geometry is profoundly affected by resonant surfaces, which give rise to non-analytic dependence of the equilibrium. As a result, the use of ideal MHD to model a hot, near collisionless 3D plasmas cannot be formally justified. As such the study of stability and Alfvén eigenmode spectrum of (m,n) field harmonics are limited and remains an open problem in 3D equilibrium fields which are typically a blend of nested magnetic surfaces, islands, and chaotic regions.

Throughout the most recent decade, the Multi-Region relaxed Magnetohydrodynamics has been developed and advances as fixed/free boundary equilibrium theory [1,2], ideal and tearing stability theory [3,4,5], to examine non-axisymmetric plasma where the magnetic islands and chaos co-exist. Theoretically, MRxMHD employs a generalization of Taylor relaxation model by using a sequence of sharp boundaries for which discontinuities in the pressure and magnetic fields are present and allows for relaxation and “tearing” at rational surfaces.

In this work, we present the Lagrangian formalism to analyze the normal mode spectra of MRxMHD for the low-amplitude short wavelength perturbation. To account for stability, we decompose the Lagrangian by considering the quadratic form of both potential and kinetic energy, and solve as a generalized eigenvalue problem. The numerical solutions are computed by upgrading the well-established Stepped Pressure Equilibrium Code which uses the mixed spectral-Galerkin representation for the vector potential to the normal modes of Stepped Pressure Equilibrium Code. Perturbed quantities are evaluated using well-posed matrix perturbation theories. In a preliminary stage, we conducted a verification study of frequency domain code, SPECN with MISHKA-1 [6] and CSCAS [7], in terms of growth rates and Alfvén eigenmodes spectrum, in an axisymmetric tokamak. Figure 1 depicts a comparison of growth rates between SPECN and MISHKA-1 as a function of Bussac beta,  $\beta_p$ , for an ideal internal kink instability. In addition, with non-uniform density profile, we are in process to examine the existence of Alfvén timescale in stepped-pressure

equilibrium solutions.

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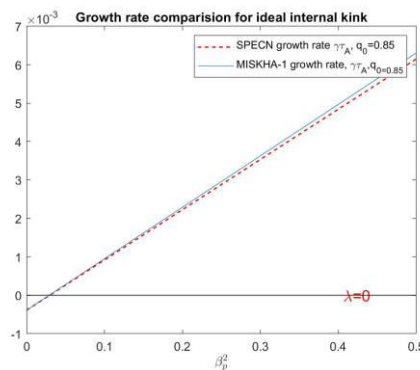


Figure 1: Comparison of growth rates between SPECN and MISHKA-1. Equilibrium parameters are considered from Bussac et al (**Phys. Rev. Lett.** 35, 1638 (1975)). Here,  $q_0 = 0.85$  denotes the value of safety factor on axis.