Small-scale and large-scale helical structures in fluid and (Hall)-MHD turbulence, and their role on the dynamics of magnetized shear flows

Annick Pouquet[1], Nobumitsu Yokoi[2],
[1] Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder,
[2] Institute of Industrial Science, University of Tokyo
pouquet@ucar.edu (speaker)

Helicity is a measure of the breakage of reflectional symmetry and a representative of topological properties of turbulent flows. It consists of links, twists and knots of field lines, and it can contribute in a crucial way to the dynamics and fundamental statistics of fluid and plasma turbulence. Helicity, in its fluid form, is the correlation between velocity and its curl, the vorticity; it is an axial scalar which can be viewed as a specific combination of shear flows. Upon integration, it has been known to be an invariant of the ideal (non-dissipative) equations since the mid-sixties, following pioneering works in MHD for magnetic helicity (correlation between the magnetic potential and its curl), and cross-helicity (correlation between the velocity and the magnetic field). These results have now been generalized, for example for two-fluid Hall MHD. We review here a few of the physical features, both old and newly-emerged, of such helical systems and of their modeling[1].

Kinetic helicity is an essential ingredient of the dynamo problem, *i.e.* the generation of large-scale magnetic fields by small-scale helical vortices. As such, it has been amply examined in the astrophysical context, as well as for plasmas for which helicity also provides strong constraints on the dynamics, such as Taylor relaxation. Cross-helicity also plays an important role in the dynamo by modifying the turbulent electromotive force, including in the strongly compressible case[2].

An intriguing result concerns the behavior of the growth of large-scale magnetic fields for two-fluid Hall-MHD systems as a function of the control parameter R (the ratio of the forcing scale to the ion inertial scale). One finds an exponential variation with R of the temporal growth rate of magnetic and generalized helicity. This result relies in a central way on the behavior of such inverse-cascade spectra: they are assumed to follow the scaling predicted by dimensional analysis, although such a spectral variation is not always observed, in particular for neutral fluids and sometimes in MHD as well[3]. An analysis of the variation of these results with (increasing) Reynolds number remains to be performed.

Furthermore, invariants provide so-called exact scaling relationships for third-order (mixed) structure and/or correlation functions, originally derived under several hypotheses: homogeneity, incompressibility, isotropy, stationarity and regularity in the presence of dissipation (with some of these hypotheses now having been relaxed). Such laws for the conservation of total energy in MHD and Hall-MHD have been observed in the Solar Wind, as well as in direct numerical simulations, providing a direct mean of evaluating both the energy dissipation rate and the direction of its cascade. Similar exact laws were derived more recently for helical invariants. These new results provide strong constraints on the realizable turbulent electromotive force, and they show as well the importance of Beltramization of vortex and current structures through weakening of the relevant forces acting on the plasma, effectively increasing the lifetime of such turbulent eddies. These helical relationships have not been sufficiently exploited up to now. They may bring an enhanced description of geophysical and astrophysical fluids as more data becomes available, for example with the Magnetospheric Multi-Scale Mission, or with the recently launched Parker Solar Probe, and for planetary magnetospheres.

We consider next how turbulent transport is affected by helical constraints, in the presence or not of an imposed large-scale shear flow, in particular in the context of magnetic reconnection and fusion plasmas. One may use theoretical tools such as the multiple-scale Direct Interaction Approximation (DIA) framework which allows to evaluate transport coefficients such as eddy viscosity, eddy diffusivity, and eddy noise[2,4]. The difference between kinetic and magnetic energy also plays a role in the dynamics. Important issues on how to construct turbulence models for non-reflectionally symmetric helical flows will also be mentioned, and the role of the cross-helicity in partially suppressing traditional transport will be emphasized.
Finally, if time permits, we shall briefly mention the intrinsic role of helicity in the development of small-scale, possibly singular, structures.

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References

[1] A Pouquet and N Yokoi, Phil. Trans. Roy. Soc. A, Special Issue: *Scaling the Turbulence Edifice*. Under revision. See also ArXiv:2104.12855 (2021)
[2] N Yokoi, J. Plasma Phys. **84**, 735840501 (2018)
[3] A Pouquet, J Stawarz and D Rosenberg, Atmosphere **11**, 203 (2020)
[4] N Yokoi and M Hoshino, Phys. Plasmas **18**, 111208 (2013)