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Adiabatic Ott-Antonsen theory for amplitude mediated metastable systems

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Dimensional reduction has been traditionally found to be a useful tool to understand correlated nonlinear dynamical systems. Such study facilitates simulating nonlinear N-body problems at a much-reduced computational cost. However, building efficient reduced dimensional models, keeping the essence of the richness of the higher dimensional phase space has always been challenging.

A system of N globally coupled phase oscillators has been found to show synchronization [1], metastability [2], and many such rich phenomena. Remarkably in 2008, Edward Ott and Thomas Antonsen observed a low dimensional behavior of such globally coupled system of phase oscillators [3]. Since then the work has been extended for a multitude of systems with different forms of the coupling between the oscillators.

However, all these previous reduced dimensional models were limited to phase oscillators only. Building a self-consistent reduced dimensional model including the amplitude dynamics, have always been hard. A straight-forward extension of the Ott-Antonsen ansatz for amplitude mediated dynamics does not reduce the integro-differential partial differential equations into an explicit finite set of nonlinear ordinary differential equations, as it did earlier for its phase-only counterpart.

In this work, we show a system of such amplitude mediated globally coupled oscillators does show low dimensional behavior, when the amplitude dynamics varies at a 'frequency' slower than its phase dynamics. The adiabatic nature of amplitude evolution allows us to separate the coupled dynamics into two timescales, the slow one for the amplitude-evolution and the fast one for the phase-evolution. The problem essentially boils down to a WKB-type analysis where the amplitude is considered as the generalized "action" variable and the phase, as the generalized "angle" variable. As an example, we take a system of N globally coupled complex Ginzburg-Landau oscillators. Previous studies showed existence of metastable states for such system of oscillators [4]. The amplitude dynamics for such states have been known to be adiabatically varying when the parameters are chosen close to the saddle-node bifurcation line [4]. We test our perturbative analysis for such nonlinear complex oscillators and find good agreement between the earlier results from high dimensional simulation and our current reduced dimensional analysis.

System of globally coupled nonlinear oscillators with self-consistent amplitude dynamics share many resemblances with fluid flows and thus are essential bits to understand fluid turbulence. The amplitude mediated metastable states are useful to understand spatio-temporal patterns found in fluid flow experiments [5,6] and in strongly coupled medium. The existence of an adiabatic metastable state and diffusion in energy has also been observed in non-equilibrium thermodynamics [7] of ergodic Hamiltonian systems that possess a slow and fast time scale.

To summarize, we have extended the Ott-Antonsen theory using WKB approach to explain the dynamics of amplitude mediated metastable systems. Our new theory assumes an adiabatic variation of the self-consistent amplitude dynamics for a system of globally coupled nonlinear oscillators. We employ a perturbative WKB analysis coupled with the celebrated Ott-Antonsen ansatz to explain the remarkable low dimensional behavior of the coupled nonlinear systems. Our study may find useful applications to understand the spontaneous emergence of spatiotemporal patterns in fluid flow experiments as well as in strongly correlated mediums.

References:

[1] Yoshiki Kuramoto. International symposium on mathematical problems in theoretical physics. Lecture notes in Physics, 30:420, 1975.
[2] Y Kuramoto and D Battogtokh. Coexistence of coherence and incoherence in nonlocally coupled phase oscillators. NONLINEAR PHENOMENA IN COM-PLEX SYSTEMS, 5(4):380–385, 2002.
[3] Edward Ott and Thomas M Antonsen. Low dimensional behavior of large systems of globally coupled oscillators. Chaos: An Interdisciplinary Journal of Nonlinear Science, 18(3):037113, 2008.
[4] Gautam C Sethia and Abhijit Sen. Chimera states: The existence criteria re-visited. Physical review letters, 112(14):144101, 2014.
[5] Dwight Barkley and Laurette S Tuckerman. Computational study of turbulent laminar patterns in couette flow. Physical review letters, 94(1):014502, 2005.
[6] Gert Brethouwer, Yohann Duguet, and Philipp Schlatter. Turbulent–laminar coexistence in wall flows with coriolis, buoyancy or lorentz forces. Journal of Fluid Mechanics, 704:137–172, 2012.
[7] Wade Hodson and Christopher Jarzynski. Energy diffusion and absorption in chaotic systems with rapid periodic driving. Physical Review Research,3(1):013219, 2021.