

# Linear Theory of Low-k Range Tearing Instability

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Magnetic reconnection is believed to be an energy conversion process from magnetic energy to plasma kinetic energy in space plasma explosions, such as solar flares and geomagnetic substorms. Tearing instability is caused by the magnetic reconnection and is believed to trigger such plasma explosions. The linear theory started from Furth, et.al. Phys.Fluids 1963 and was largely improved by Loureiro, et.al., Phys.Plasmas 2007. The former and latter theories are respectively called FKR and LSC theories. Then, the LSC theory was modified by Shimizu, AAPPS-DPP2018, and extended by Shimizu, AAPPS-DPP2021. However, LSC theory is only applicable for “high-k (wave number >1)” range. In this paper, the modified LSC theory is moreover extended to “low-k” range. In this paper, the perturbation equations applicable for “low-k” range are solved, as follows.

$$\nu \phi'''' / (\kappa \epsilon^2) = (\lambda + 2\kappa\nu)\phi'' - (\lambda + \kappa\nu)\kappa^2 \epsilon^2 \phi - \xi \phi''' / \kappa + \kappa \epsilon^2 (2\phi + \xi \phi') + \kappa f(\xi)(\lambda \psi - f(\xi)\phi - \xi \psi' / \kappa) - f''(\xi)\psi, \quad (1)$$

$$\lambda \kappa \psi = \xi \psi' + \kappa f(\xi)\phi + \psi'' - \kappa^2 \epsilon^2 \psi, \quad (2)$$

Eq.(2) is essentially the same as Eq.(7) derived by Loureiro (2007) where  $\phi$ ,  $\psi$ , and  $f$  are respectively the perturbed plasma flow function, perturbed magnetic field function, and the equilibrium magnetic field function. Loureiro did not directly solve Eq.(2). Modifying Eqs.(1)-(2), they approximately solved only “large-k” range. Also, Eq.(1) is the same as Eq.(6) in the paper but viscosity  $\nu$  terms are newly introduced, here. Excepting  $\nu$ , every notation is the same as that of Loureiro (2007). The image of the modified LSC theory is drawn in Figure 1, where  $\xi c$  is defined to be the location of the upstream “open” boundary. It is important that low-k range can be solved by introducing the viscosity  $\nu$ .

Shimizu (2018) showed that the perturbation equations derived by Loureiro (2007) can be numerically solved as an initial value problem from the origin  $\xi = 0$ . Then, Shimizu (2021) also showed that the viscosity can be extensively introduced in the equations. Then, Eqs.(1)-(2) for low-k range can be solved by introducing the viscosity effect, as shown below. Figure 2 shows one of the zero-crossing solutions numerically solved for  $\phi = \psi = 0$  and  $\phi' = -1.8$  at  $\xi = \xi c$ . At this point, zero-contact solution, i.e.,  $\phi = \psi = 0$  and  $\phi' = 0$  at  $\xi = \xi c$ , is not found unlike large-k case (e.g., AAPPS-DPP2021). It suggests that the viscosity effect essentially needs the field aligned flow, i.e., non-zero  $\phi'$ , along the upstream boundary to cause the instability, as shown in Figure 1.

Figure 3 shows how the linear growth rate  $\lambda$  varies for  $\xi c$  and wave number  $k$ , where resistivity  $e=0.1 (= \epsilon)$  and  $N=0.001 (= \nu)$ . Basically, as  $\xi c$  increases,  $\lambda$

increases. This tendency is the same as previous cases (AAPPS-DPP,2018 and 2021) but should be noted that this is non-zero  $\phi'$  upstream condition, unlike AAPPS-DPP2021. It is the most remarkable that the  $\lambda$  curve of  $k=0.2$ , i.e., low-k range, starts from 17.75. At the point, when  $\phi'$  at  $\xi = \xi c$  is close to zero, these  $\lambda$  curves totally shift to large  $\xi$ . It suggests that tearing instability is stabilized in low-k range. In previous works (e.g., FKR), it is known that  $k \epsilon = c$  (a constant determined for upstream conditions and so on) is a criterion in large-k range. This paper suggests that another criterion exists in low-k range, at least, when  $\xi c$  is finite.

## References:

- T.Shimizu, A New Viewpoint for Linear Theory of Tearing Instability, AAPPS-DPP, 2018Nov., SGP-04.  
T.Shimizu, Linear Theory of Viscous-Resistive Tearing Instability, AAPPS-DPP, 2021Sept., SG-I41.

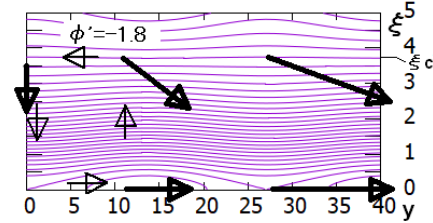


Figure 1: Schematic Image of Tearing Instability in LSC. Thick and thin arrows respectively show the equilibrium of plasma flow field and perturbed flow field. The latter is only shown around the origin.

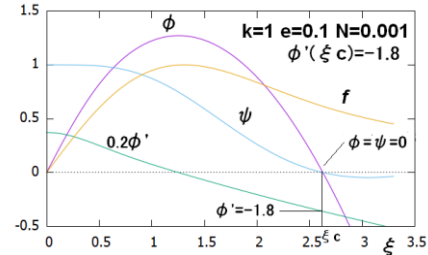


Figure 2: An example of zero-crossing solution at  $\lambda = 0$  where  $\phi = \psi = 0.0$  and  $\phi' = -1.80$  at  $\xi c = 2.626$ .

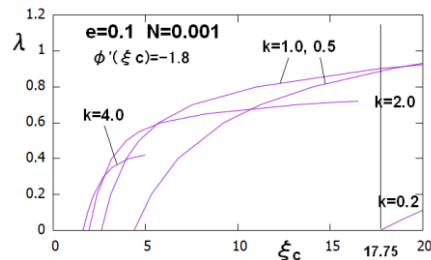


Figure 3: The linear growth rate  $\lambda$  versus  $\xi c$  of the zero-crossing solutions for some  $k$ , where the low-k range ( $k < 1$ ) is  $k=0.2$  and  $0.5$ .