6th Asia-Pacific Conference on Plasma Physics, 9-14 Oct, 2022, Remote e-conference

Recovering Gardner Restacking with Purely Diffusive Operations

E. J. Kolmes[1] and N. J. Fisch[1]

[1] Department of Astrophysical Sciences, Princeton University

e-mail (speaker): ekolmes@princeton.edu

The total amount of energy that could possibly be liberated from a system – that is, the free or available energy – is a quantity of interest in a variety of plasma systems. How much energy is accessible in a particular system is determined by the set of operations that can be performed on the system; each distinct set of operations will be associated with a free energy.

One of the first approaches to this problem, undertaken by C. S. Gardner, defines a set of operations (and a corresponding free energy) as any operation that conserves phase space volumes [1]. The operations are sometimes called Gardner restacking; the free energy is sometimes called the Gardner free energy.

Another major approach, originally proposed to model the kind of phase space mixing associated with wave-particle interactions, is to allow any operation that tends to equalize the populations of pairs of elements in phase space [2]. The resulting free energy is called the diffusively accessible free energy.

Either set of operations can be restricted to respect one or more conservation laws [3-5]. These restricted free energies may be useful for predicting instability levels in real experiments [6].

For any discrete system, with some finite number of phase space elements, the Gardner free energy exceeds the diffusively accessible free energy (except in cases where both vanish) [7]. However, it was recently shown that for continuous systems, these free energies are – surprisingly enough – equal [8, 9].

This talk will discuss the spectrum of states that are accessible through diffusive operations, and how best to understand the relationship between these different notions of free energy.

[1] C. S. Gardner, *Bound on the energy available from a plasma,* Phys. Fluids **6**, 839 (1963).
[2] N. J. Fisch and J.-M. Rax, *Free energy in plasmas under wave-induced diffusion,* Phys. Fluids B **5**, 1754 (1993).
[3] P. Helander, *Available energy and ground states of collisionless plasmas,* J. Plasma Phys. **83**, 715830401 (2017).
[4] P. Helander, *Available energy of magnetically confined plasmas,* J. Plasma Phys. **86**, 905860201 (2020).
[5] E. J. Kolmes, P. Helander, and N. J. Fisch, *Available energies from diffusive and reversible phase space rearrangements,* Phys. Plasmas **27**, 062110 (2020).
[6] R. J. J. Mackenbach, J. H. E. Proll, and P. Helander, *Available energy of trapped electrons and its relation to turbulent transport,* Phys. Rev. Lett. **128**, 175001 (2022).
[7] M. J. Hay, J. Schiff, and N. J. Fisch, *Maximal energy extraction under discrete diffusive exchange,* Phys. Plasmas **22**, 102108 (2015).
[8] E. J. Kolmes and N. J. Fisch, *Recovering Gardner restacking with purely diffusive operations,* Phys. Rev. E **102**, 063209 (2020).
[9] E. J. Kolmes, *Particle, Charge, and Energy Rearrangement in Rotating Magnetized Plasma,* Ph.D. thesis, Princeton University (2022).