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High order numerical methods for a hybrid fluid-kinetic plasma model

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In this talk, we focus on the numerical approximation of a hybrid fluid-kinetic plasma model for electrons, in which energetic electrons are described by a Vlasov type model whereas a fluid model is used for the cold population of electrons. The two models are coupled through the current in the Maxwell equations for the electromagnetic fields. Such hybrid modelling is of paramount importance to describe multiscale phenomena arising in Earth magnetosphere or tokamak plasmas in which hot particles interact with a cold bulk. The stiffest physical scale is indeed removed in this modelling, which enables to use numerical parameters that are not constrained by the stiffest physical scale.

For this hybrid model, we discuss two reformulations which will serve as a basis for our numerical purposes. First, a Hamiltonian structure is proposed (with a Poisson bracket and a total energy) whereas in a second approach, we separate the linear and nonlinear terms of the model.

The goal of this work is twofold. First, we investigate numerically the validity of the hybrid model compared to a fully Vlasov model, which contains a stiff physical scale (here the temperature cold/hot ratio). This is done in a reduced phase space configuration (one dimension in space and one dimension in velocity). Second, a comparison of Eulerian solvers (which use a grid of the phase space) is discussed in a four dimensional phase space (one dimension in space and three dimensions in velocity), on a Weibel type instability.

To do so, we consider two Eulerian numerical methods, which use high order numerical approximation on a grid of the phase space.

The first one is based on the Hamiltonian structure of the hybrid model which enables us to design a Hamiltonian splitting. It turns out that each subsystem generated by the Hamiltonian splitting can be solved exactly in time, which means that the time error only comes from the splitting. This approach are known to provide very good conservation of the total energy for large time, which is of particular importance for plasma applications. However, even if this approach turns out to be efficient in low dimensional case, its computation cost becomes a drawback when high order methods in time or when high dimensional configurations are employed since in both cases, the number of stages of the splitting increases dramatically.

The second approach we propose here is based on linear-nonlinear splitting reformulation amenable to use the so-called exponential time integrators. These methods are based on a variation of constant formula where the linear part of the model is solved exactly in time whereas the nonlinear part is approximated by Runge-Kutta type strategies. This approach enables to derive high order time integrators still removing the CFL condition induced by the linear part (which is the most stringent one in practice), and whose cost is linear with respect to the order of the numerical method. We also propose a Padé approximation technique to approximate the exponential of the matrix corresponding to the linear part of the model ; this Padé approach enables to preserve the pure imaginary spectrum of the continuous model which has important consequences for the stability of the numerical method and thus allows to take large time steps.

The accuracy and efficiency of these two methods, which are both combined with an adaptive time stepping strategy, are discussed on a Weibel instability test case. During the linear phase in which the nonlinear effects are small, the exponential method is able to consider large time steps whereas, in the nonlinear phase, the method automatically takes smaller time steps to ensure the nonlinear stability of the method.

References

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