

Existence of weakly quasisymmetric magnetic fields in asymmetric toroidal domains with non-tangential quasisymmetry

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A quasisymmetry is a special symmetry that enhances the ability of a magnetic field to trap charged particles: in principle, a carefully designed quasisymmetric magnetic field would provide the field line twist needed to confine charged particles within a bounded domain only through a set of suitably crated external coils. For this reason, quasisymmetric magnetic fields and their generalizations are at the basis of the design of next-generation fusion reactors (stellarators) that would operate in a condition close to a steady state. This is in contrast with an axially symmetric tokamak, which relies on a self-sustained toroidal plasma current to generate the rotational transform, making the system prone to instabilities.

However, the existence of quasisymmetric magnetic fields has remained an unsolved problem due to the complexity of the governing first-order nonlinear partial differential equations, which include the system boundary as unknown variable.

In this study, we exploit an analytical method (Clebsch parametrization) to construct, for the first time, a family of weakly quasisymmetric magnetic fields [1,2]. The obtained solutions hold in a toroidal volume, are smooth, possess nested flux surfaces, are not invariant under continuous Euclidean isometries (translations, rotations, and their combination), have a non-vanishing current, exhibit a vanishing rotational transform, and fit within the framework of anisotropic magnetohydrodynamics. Due to the vanishing rotational transform, these solutions are however not suitable for particle confinement.

An example of quasisymmetric configuration without rotational transform obtained with the method outlined above is the following:

$$\begin{aligned} \mathbf{B} &= \nabla \left[z - \epsilon \sin \left(m\varphi + \frac{z}{r} \right) \right] \times \nabla \log r, \\ \mathbf{u} &= -\frac{1}{3} \nabla \left(m\varphi + \frac{z}{r} \right) \times \nabla r^3, \\ \Psi &= \frac{1}{2} \left\{ (r - r_0)^2 + \alpha \left[z - \epsilon \sin \left(m\varphi + \frac{z}{r} \right) \right]^2 \right\}. \end{aligned}$$

Here, \mathbf{B} denotes the magnetic field, \mathbf{u} the direction of quasisymmetry, Ψ the flux function with toroidal level sets, ϵ, α positive constants, m an integer, and (r, φ, z) cylindrical coordinates. A plot of this configuration is given in figure 1. Observe that the

quasisymmetry \mathbf{u} is not tangential to the flux surfaces Ψ . This is the reason why the conserved momentum arising from the quasisymmetry is $p \approx r$, instead of $p \approx \Psi$ as desirable for a stellarator. In particular, this implies that particles are only radially confined, making this quasisymmetric configuration unsuitable for plasma confinement.

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References

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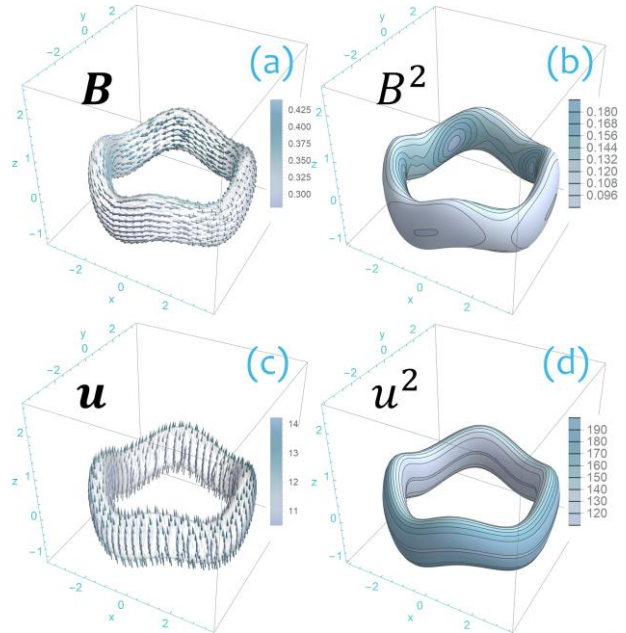


Figure 1. An example of quasisymmetric configuration without rotational transform. (a) Magnetic field \mathbf{B} , (b) magnetic field strength B^2 , (c) quasisymmetry \mathbf{u} , and (d) modulus u^2 over a flux surface Ψ .