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Flux inequality and dual cascade process in two-dimensional and geostrophic turbulence

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The dual cascade process, in which energy and enstrophy are simultaneously transferred to the lower and larger wavenumbers, respectively, in the wavenumber space, is a well-known fundamental property of forced-dissipative turbulence governed by two-dimensional (2-D) Navier-Stokes (NS) equation. Although the existence of the dual cascade process has been known since the 1960s, its mathematical proof from 2-D NS equation has remained an unsolved problem. Gkioulekas and Tung[1] succeeded in proving the existence of the dual cascade process by combing the classical theory of 2-D turbulence[2] and an inequality between the energy flux $\Pi_E(k)$ and the enstrophy flux $\Pi_Z(k)$,

$$k^2\Pi_E(k) - \Pi_Z(k) < 0, \qquad (1)$$

where k is a wavenumber. Because the inequality (1) is originally proposed and proven by Danilov, it is often referred to as Danilov's inequality. The inequality is valid not only in the energy and enstrophy inertial ranges but also in all wavenumber ranges except forcing wavenumber ranges. Furthermore, Danilov's inequality can be extended as

$$k^\alpha\Pi_E(k) - \Pi_Z(k) < 0, \qquad (2)$$

to the case of a generalized two-dimensional fluid system, the so-called α-turbulence system,

$$\frac{\partial}{\partial t} q = -J(\psi, q) + D + F,$$
$$(-\nabla^2)^{\alpha/2}\psi = q.$$

Here, α is a real number, J is the horizontal Jacobian and q and ψ are the generalized vorticity and the stream function, respectively. D and F represent dissipation and forcing terms, respectively. Using (2), it is possible to prove the existence of dual cascade process in α-turbulence. Thus, Danilov's inequality can be interpreted as a fundamental relation for the family of two-dimensional turbulence.

In the present study, we numerically investigate whether the inequality (1) holds for well-known geophysical quasi 2D fluid system, two-layer quasi-geostrophic (QG) potential vorticity equations.[3] The governing equation of this system is

$$\frac{\partial}{\partial t} q_j = -J(\psi_j, q_j) + (-1)^j U \frac{\partial}{\partial x}(q_j + k_\mathrm{d}^2\psi_j)$$
$$+(-1)^{\gamma-1}\nu\nabla^{2\gamma}q_j - \kappa\,\delta_{2j}\nabla^2\psi_j, \qquad (3a)$$

$$q_j = \nabla^2\psi_j + \frac{k_\mathrm{d}^2}{2}(\psi_{3-j} - \psi_j), \quad (j=1, 2). \qquad (3b)$$

Here, the subscripts 1 and 2 refer to the upper and lower layers, respectively. q and ψ are the potential vorticity and the stream function, respectively. The quantity U in the second term on the right-hand side of (3a) is the velocity difference between the upper and lower layers. The second term on the right-hand side of (3a) acts as forcing through the baroclinic instability. The last term in (3a) represents the bottom friction. For Danilov's inequality of the two-layer QG system, the quantities Π_E and Π_Z in (1) should be interpreted as the total energy flux and the potential enstrophy flux. Danilov's inequality of this system is mathematically sign indefinite due to the presence of forcing term and bottom friction term.[4] The possibility of the breakdown of Danilov's inequality was discussed in [4]. Therefore, we investigate the validity of Danilov's inequality of the two-layer QG system by comprehensive numerical simulations of (3).

The characteristics of solution of (3) are mainly controlled by a non-dimensional parameter $2Uk_\mathrm{d}/\kappa$, which is referred to as throughput. When the throughput is small, the total energy is dominated by the potential energy. In contrast, when the throughput is large, the total energy is dominated by the kinetic energy. We performed numerical simulations of (3) for wide throughput range, $O(10^{-2}) < 2Uk_\mathrm{d}/\kappa < O(10)$, paying careful attention to the budgets of energy and enstrophy and the resolution of the simulations. We found no evidence of the violation of Danilov's inequality across the whole wavenumber range (see Figure 1).

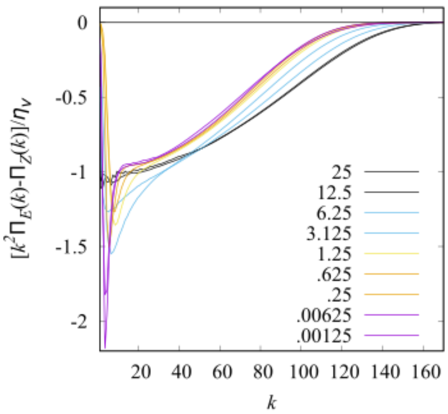

Figure 1: Flux differences $k^2\Pi_E(k) - \Pi_Z(k)$ for various values of the throughput. Ordinate is normalized by the potential enstrophy dissipation rate.

References

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