Lagrangian point-models for unstable interfaces: outdated technique or modernisable approach?

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Describing the dynamics of an unstable interface between two inviscid fluids is a long-standing problem in fluid dynamics; the problem encapsulates an incredibly diverse array of behavioural regimes and thwarts smooth, unique solutions in all but the simplest cases. For plasma physics, understanding this interface motion is relevant for modelling a number of important phenomena, particularly the generation of hotspots in inertial confinement fusion (ICF).

Presently, direct numerical simulation by the finite volume method have come to be the preferred technique for most fluid dynamics problems including interface dynamics [1]. The finite volume method uses the divergence theorem to simplify conservation equations to a system of coupled, cell-based boundary value problems, which can easily be solved using a Riemann solver. The method is both powerful and popular due to its ease of implementation, speed of computation and the fact it is easily adapted to a broad range of problems. However, the method has drawbacks. The resolutions required to resolve large, multiscale problems is prohibitively expensive; shocks and steep gradients are often smoothed or diffused, despite the best efforts of sophisticated Riemann solvers; and the method has harsh limits on its ability to account for nonlinear and nonlocal effects [2].

Prior to the development of the computational power necessary to use these methods, however, the prevailing technique involved deriving a reduced system of Lagrangian equations for the interface directly from Eulerian conservation equations and discretising them [3]. This method is attractive from a mathematical point of view, but the naive discretisations commonly applied are prone to numerical instabilities, and often have unknown convergence [4].

We revisit this historical approach by considering the problem of two inviscid, incompressible fluids accelerated against their density gradient. The relevant conservation equations governing the instability are the incompressible Euler equations, for which Lagrangian interface equations already exist. To solve these, we propose a robust method that uses operator theory to reduce the problem to a system of first order equations treatable with any ordinary differential equation software package.

References

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